

Actus Specification

June 7, 2026

1 Overview

This blueprint is a machine-checked re-authoring of the *ACTUS Technical Specification* — the algorithmic standard for representing financial contracts by their cash flows. It reproduces the standard’s normative content (notations, utility functions, observers, and the per-contract schedule, state-initialisation, state-transition and payoff tables) and links every definition to a Lean 4 formalization, so that the document is at once the specification and its proof of internal consistency.

It covers all eighteen contract types of the standard (§5). Every *stateful* contract — the lending family (PAM, LAM, NAM, ANN, CLM) together with the exotic amortizer LAX, the plain-vanilla swap SWPPV and the non-maturity deposit UMP — is modeled twice: relationally (the inductive **Step**, the readable spec) and functionally (computable state-transition and payoff functions), with the two proven to agree. The remaining types — positions, derivatives, composite swaps (SWAPS), and credit enhancement — are direct cash-flow builders over the shared schedule-generation engine.

The whole development is generic over the amount type and instantiated twice: at `Float` for the executable engine — validated against the ACTUS reference test suite, where it reproduces all 236 gated reference contracts (3155 cash flows) exactly — and at the real numbers \mathbb{R} for the quantitative metatheorems of §6. The order/field structure those bounds need exists over \mathbb{R} but not over `Float` (where `NaN` even breaks $a \leq a$), so they are genuine theorems over \mathbb{R} rather than empirical checks.

Provenance. The normative formulas and tables are derived from the ACTUS Technical Specification, © 2018–present ACTUS Financial Research Foundation, distributed under the Creative Commons Attribution–ShareAlike 4.0 licence; this re-authoring is a derivative work under the same licence.

2 Notations

2.1 Contract attributes

Contract *attributes* are the legal contractual terms that define a contract’s exchange of cash flows; they are catalogued in the ACTUS data dictionary and referenced here by their dictionary short names in `typewriter` font, e.g. `IED` for the *initial exchange date*. Vector-valued attributes are indexed with a subscript: the i -th element of `ARPRANX` is `ARPRANXi`.

2.2 The \emptyset -operator

The symbol \emptyset marks a quantity as *undefined*: an optional attribute that carries no value, or a schedule that contains no times ($\overline{t}^{IP} = \emptyset$ is the empty schedule). In the Lean development this is the `Option` type; an absent term is `none`.

2.3 t_0 -time and state variables

t_0 denotes the contract’s status date `SD` — the time as of which its terms and implied state are represented; the contractual logic determines events and states for all $t > t_0$. *State variables* describe the contract state at a time t ; they are written capitalised and bold and indexed by time, e.g. \mathbf{Nt}_t is the outstanding notional principal at t . The Lean counterpart is a `State` record carrying `Nt`, `Ipnr`, `Ipac`, `Feac`, `Nsc`, `Isc`, `Sd`, etc.

2.4 Contract events, STF and POF

A contract *event* e_t^k is a scheduled or unscheduled occurrence at time t of type k , carrying an event time and a payoff $c \in \mathbb{R}$, retrieved by $\tau(\cdot)$ and $f(\cdot)$ respectively. Two functions act on an event: the *state-transition function* `STF_k_CT()` maps the pre-event state to the post-event state, and the *payoff function* `POF_k_CT()` yields the cash flow $f(e_t^k)$. Both are specific to an event type k and a contract type CT . In Lean these are the per-contract `stf` and `pof` functions; the relational `Step` is the graph of `stf`.

Plain/primed state in a transition. The per-contract tables of Section 5 use a light convention in place of the standard’s t^-/t^+ time decorations: a *plain* state symbol denotes its pre-event value (written \cdot , the standard’s \cdot_{t^-}) and a *primed* symbol its post-event value (\cdot' , the standard’s \cdot_{t^+}). An assignment such as $\mathbf{Nt}' = \mathbf{Nt} - \dots$ then reads “new notional = old notional – ...”; a primed symbol may also appear on the right-hand side when an update depends on a value already revised earlier in the same event. For the interest-bearing contracts we further abbreviate the interest accrued on the outstanding notional since the last state date by

$$\Delta\mathbf{Ipac} := Y(\mathbf{Sd}, t) \cdot \mathbf{Ipnr} \cdot \mathbf{Nt},$$

the year fraction from the last state date `Sd` to the event time t times the nominal rate `Ipnr` times the notional `Nt`; this term recurs in almost every transition below.

2.5 Event sequence and contract lifetime

Events occurring at the same time are ordered by a fixed event-sequence indicator, so that their STFs and POFs compose deterministically. A contract’s *lifetime* runs from `SD` to $\min(\text{MD}, \text{AMD}, \text{PR}^*, \text{STD}, \text{TD}, t^{\text{max}})$, where PR^* is the principal-redemption event that drives `Nt` to 0; terms that do not apply to a given contract are taken as $+\infty$, and t^{max} optionally bounds the analysis horizon for contracts without a natural end (e.g. `STK`).

3 Utility functions

The standard is built on a small layer of *utility functions* shared by every contract. The development is generic over the amount type α (an ordered field for the metatheorems, `Float` for the engine).

Definition 1 (Schedule function S). The schedule function maps an anchor s , a cycle c and an end date T to the cyclic sequence

$$S(s, c, T) = \vec{t} = \begin{cases} \{\} & \text{if } s = \emptyset \wedge T = \emptyset \\ s & \text{else if } T = \emptyset \\ (s, T) & \text{else if } c = \emptyset \\ (s = t_1, \dots, t_n = T) & \text{else} \end{cases}$$

with $t_i < t_{i+1}$. Cycles have the form NPS (N an integer, $P \in \{D, W, M, Q, H, Y\}$ a period unit, $S \in \{+, -\}$ a stub rule). An optional boolean B in $S(s, c, T, B)$ controls whether the end date T belongs to the schedule, and the **EOMC/BDC** conventions (below) may shift the resulting times, so the sequence need not be equidistant.

Definition 2 (Array schedule \vec{S}). Array schedules generalise S to vector inputs $\vec{s} = (s_0, \dots, s_m)$, $\vec{c} = (c_0, \dots, c_m)$:

$$\vec{S}(\vec{s}, \vec{c}, T) = (S(s_0, c_0, s_1 - c_0), \dots, S(s_m, c_m, T)),$$

coinciding with S for $m = 1$. Used by LAX (§5.3).

Definition 3 (End-of-month and business-day conventions). The end-of-month convention **EOMC** fixes whether cyclic times anchored on a short month-end fall on the same day or the month-end; the business-day convention **BDC** shifts a time off a non-business day (following/preceding, modified or not, calculate-before or -after the shift); and the business-day calendar $B : t \mapsto \{\text{true, false}\}$, selected by **CLDR**, decides which days are business days (**NoHoliday**, **MondayToFriday**, ...).

Definition 4 (Year-fraction convention Y). The year-fraction interface $Y : s, t, \text{DCC} \mapsto \mathbb{R}$ gives the fraction of a year between two times under a day-count convention **DCC** (Actual/360, Actual/365, 30E/360, Actual/Actual ISDA, Business/252). Per-annum rates are accrued by multiplying with Y .

Definition 5 (Contract-role sign R). The contract-role sign $R : \text{CNTRL} \rightarrow \{-1, +1\}$ maps a contract role to +1 for a claim (asset) or -1 for an obligation, fixing the direction of every role-sensitive cash flow and state (a positive flow runs from **CPID** to **CRID**). E.g. **RPA** $\mapsto +1$, **RPL** $\mapsto -1$, **GUA** $\mapsto -1$, **OBL** $\mapsto +1$. We abbreviate $R(\text{CNTRL})$.

Definition 6 (Annuity amount A). The annuity amount sizes the constant **ANN** instalment so the notional is fully repaid by maturity:

$$A(s, T, n, a, r) = (n + a) \frac{\prod_{i=1}^{m-1} 1 + rY(t_i, t_{i+1})}{1 + \sum_{i=1}^{m-1} \prod_{j=i}^{m-1} 1 + rY(t_j, t_{j+1})},$$

with a the accrued interest at s , r the rate, and t_1, \dots, t_m the future PR-schedule times of the annuity (§5.5).

Definition 7 (Canonical payoff and settlement FX). The canonical payoff of a contract aggregates all future event payoffs under frozen risk-factor conditions, $F(x, t) = \sum_{c \in C} c$. When a contract settles in a currency **CURS** other than its denomination **CUR**, payoffs are scaled by the settlement FX rate

$$X_{\text{CUR}}^{\text{CURS}}(t) = \begin{cases} 1 & \text{if } \text{CURS} = \emptyset \vee \text{CURS} = \text{CUR} \\ O^{rf}(\text{CUR}/\text{CURS}, t) & \text{else,} \end{cases}$$

which appears as the leading factor of every payoff function below.

4 Composition and observers

Definition 8 (Contract composition). Combined contracts derive their payoff from *child contracts* (underlyings) referenced through the attribute `CTST`, a list of contract references each carrying an `object`, a `type` and a `role`. We write $\text{CTST}_{\text{role}}^{\text{type}}$ to query a referenced child (omitting the index of the first match), e.g. $\text{CTST}_{\text{FirstLeg}}^{\text{Contract}}$ for a swap’s first leg or $\text{CTST}_{\text{Underlying}}^{\text{Contract}}$ for an option’s underlying.

Definition 9 (Risk-factor observer O). The risk-factor observer is the interface through which a contract reads its uncertain environment. In rate mode it returns a market state, $O^{rf} : i, t \mapsto \mathbb{R}$, and in event mode it returns observed unscheduled events, $O^{ev} : i, k, t \mapsto \{e_t^k, \dots\}$ — e.g. $O^{rf}(\text{RRM0}, t)$ for a reset reference rate, or $O^{ev}(\text{CID}, \text{PP}, t)$ for observed prepayments. The interface fixes structure only; implementations encode whatever assumption a forward-looking analysis requires.

Definition 10 (Child-contract observer U). The child-contract observer lets a parent read its children: future events $U^{ev} : i, t, a \mapsto \{e_v^k, \dots\}$, a child state variable $U^{sv} : i, t, x, a \mapsto \mathbb{R}$, or a child attribute $U^{ca} : i, x \mapsto y$ — the basis for evaluating swaps, options and credit enhancement over their underlyings.

5 Contract types

Each contract type is specified by three normative tables — its *contract schedule* (which events occur and when), its *state-variable initialisation* (the state at t_0), and its *state-transition and payoff functions* (the effect of each event) — reproduced below and linked to the Lean `stf/pof/Step` definitions that formalize them.

5.1 PAM: Principal at Maturity

PAM is the archetypal ACTUS contract and the building block of the whole maturity family. The entire principal (notional) is advanced at the initial exchange date `IED` and returned in a single bullet repayment at maturity `MD`; in between, interest accrues continuously and is paid out on the interest-payment schedule `IP`. No principal is amortized during the term, so the outstanding notional `Nt` stays constant between `IED` and `MD` — which makes PAM the natural model for fixed-term bonds, bullet loans and term deposits.

The contract state carries the outstanding notional `Nt`, the nominal rate `Ipnr`, accrued interest `Ipac` and fees `Feac`, and the notional/interest scaling indices `Nsc/Iscl`. Optional features layer on through their own events: rate resets (`RR/RRF`, optionally clamped to the life window [`RRLF, RRLC`]), index scaling (`SC`), periodic fees (`FP`), prepayment penalties (`PY`), unscheduled prepayment (`PP`) and early termination (`TD`). Because every other lending contract (`LAM, NAM, ANN, CLM`) is specified as PAM plus a redemption rule, the PAM state-transition and payoff functions below are reused throughout the family.

Definition 11 (PAM state transition and payoff). The PAM state-transition function $\text{STF}_{k_PAM}()$, one branch per event type, together with the payoff function $\text{POF}_{k_PAM}()$, as given by the table below.

PAM
State Variables

Variabl	Initialization per t_0		
Md	$\text{Md}_{t_0} = \text{MD}$		
Nt	$\text{Nt}_{t_0} = \begin{cases} 0.0 & \text{if } \text{IED} > t_0 \\ R(\text{CNTRL}) \times \text{NT} & \text{else} \end{cases}$		
Ipnr	$\text{Ipnr}_{t_0} = \begin{cases} 0.0 & \text{if } \text{IED} > t_0 \\ \text{IPNR} & \text{else} \end{cases}$		
Ipac	$\text{Ipac}_{t_0} = \begin{cases} 0.0 & \text{if } \text{IPNR} = \emptyset \\ \text{IPAC} & \text{else if } \text{IPAC} \neq \emptyset \\ Y(t^-, t_0) \times \text{Nt}_{t_0} \times \text{Ipnr}_{t_0} & \text{else} \end{cases}$ with $t^- = \sup t \in \bar{t}^{IP} \mid t < t_0$		
Feac	$\text{Feac}_{t_0} = \begin{cases} 0.0 & \text{if } \text{FER} = \emptyset \\ \text{FEAC} & \text{else if } \text{FEAC} \neq \emptyset \\ Y(t^{FP-}, t_0) \times \text{Nt}_{t_0} \times \text{FER} & \text{else if } \text{FEB} = \text{'N'}$ $\frac{Y(t^{FP-}, t_0)}{Y(t^{FP-}, t^{FP+})} \times \text{FER} & \text{else}$		
	with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$		
Nsc	$\text{Nsc}_{t_0} = \begin{cases} \text{SCI XSD} & \text{if } \text{SCEF} = \text{'[x]N[x]'} \\ 1.0 & \text{else} \end{cases}$		
Isc	$\text{Isc}_{t_0} = \begin{cases} \text{SCI XSD} & \text{if } \text{SCEF} = \text{'I[x][x]'} \\ 1.0 & \text{else} \end{cases}$		
Prf	$\text{Prf}_{t_0} = \text{PRF}$		
Sd	$\text{Sd}_{t_0} = t_0$		
Events			
Event	Schedule	Payoff Function	State Transition
AD	$\bar{t}^{AD} = (t_0, t_1, \dots, t_n)$ With $t_i, i = 1, 2, \dots$ a custom input	0.0	$\text{Ipac}' = \text{Ipac} + \Delta \text{Ipac}$ $\text{Sd}' = t$
IED	$t^{IED} = \text{IED}$	$X_{\text{CUR}}^{\text{CURS}}(t) R(\text{CNTRL})(-1)(\text{NT} + \text{PDIED})$	$\text{Nt}' = R(\text{CNTRL})\text{NT}$ $\text{Ipnr}' = \begin{cases} 0.0 & \text{if } \text{IPNR} = \emptyset \\ \text{IPNR} & \text{else} \end{cases}$ $\text{Ipac}' = \begin{cases} \text{IPAC} & \text{if } \text{IPAC} \neq \emptyset \\ y \text{Nt}' \cdot \text{Ipnr}' & \text{if } \text{IPANX} \neq \emptyset \wedge \text{IPANX} \neq \emptyset \\ 0.0 & \text{else} \end{cases}$ $\text{Sd}' = t$ with $y = Y(\text{IPANX}, t)$
MD	$t^{MD} = \text{Md}_{t_0}$	$X_{\text{CUR}}^{\text{CURS}}(t)(\text{Nsc} \cdot \text{Nt} + \text{Isc} \cdot \text{Ipac} + \text{Feac})$	$\text{Nt}' = 0.0$ $\text{Ipac}' = 0.0$ $\text{Feac}' = 0.0$ $\text{Sd}' = t$

PP	$\bar{t}^{PP} = \begin{cases} \emptyset & \text{if PPEF} = \text{'N'} \\ (\bar{u}, \bar{v}) & \text{else} \end{cases}$ <p>where $\bar{u} = S(s, \text{OPCL}, T^{MD})$ $\bar{v} = O^{ev}(\text{CID}, \text{PP}, t)$ with $s = \begin{cases} \emptyset & \text{if OPANX} = \emptyset \wedge \text{OPCL} = \emptyset \\ \text{IED} + \text{OPCL} & \text{else if OPANX} = \emptyset \\ \text{OPANX} & \text{else} \end{cases}$</p>	$X_{\text{CUR}}^{\text{CURS}}(t) f(O^{ev}(\text{CID}, \text{PP}, t))$	$\begin{aligned} \mathbf{Ipac}' &= \mathbf{Ipac} + \Delta \mathbf{Ipac} \\ \mathbf{Nt}' &= \mathbf{Nt} - f(O^{ev}(\text{CID}, \text{PP}, t)) \\ \mathbf{Sd}' &= t \end{aligned}$
PY	$\bar{t}^{PY} = \begin{cases} \emptyset & \text{if PYTP} = \text{'O'} \\ \bar{t}^{PP} & \text{else} \end{cases}$	$\begin{aligned} &X_{\text{CUR}}^{\text{CURS}}(t) R(\text{CNTRL}) \text{PYRT} \\ &c \text{PYRT} \\ &c \max(0, \mathbf{Ipnr} - O^r f(\text{RRMO}, t)) \\ &\text{with } c = X_{\text{CUR}}^{\text{CURS}}(t) R(\text{CNTRL}) Y(\mathbf{Sd}, t) \mathbf{Nt} \end{aligned}$	$\begin{aligned} \mathbf{Ipac}' &= \mathbf{Ipac} + \Delta \mathbf{Ipac} \\ \mathbf{Sd}' &= t \end{aligned}$
FP	$\bar{t}^{FP} = \begin{cases} \emptyset & \text{if FER} = \text{'A'} \\ S(s, \text{FECL}, T^{MD}) & \text{else} \end{cases}$ <p>with $s = \begin{cases} \emptyset & \text{if FEANX} = \emptyset \wedge \text{FECL} = \emptyset \\ \text{IED} + \text{FECL} & \text{else if FEANX} = \emptyset \\ \text{FEANX} & \text{else} \end{cases}$</p>	$\begin{aligned} &R(\text{CNTRL})c \\ &c \mathbf{Sd} + \mathbf{Nt} + \mathbf{Feac} \\ &\text{with } c = X_{\text{CUR}}^{\text{CURS}}(t) \text{FER} \end{aligned}$	$\begin{aligned} \mathbf{Ipac}' &= \mathbf{Ipac} + \Delta \mathbf{Ipac} \\ \mathbf{Feac}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$
PRD	$t^{PRD} = \text{PRD}$	$X_{\text{CUR}}^{\text{CURS}}(t) R(\text{CNTRL})(-1)(\text{PPRD} + \mathbf{Ipac} + \Delta \mathbf{Ipac})$	$\begin{aligned} \mathbf{Ipac}' &= \mathbf{Ipac} + \Delta \mathbf{Ipac} \\ \mathbf{Sd}' &= t \end{aligned}$
TD	$t^{TD} = \text{TD}$	$X_{\text{CUR}}^{\text{CURS}}(t) R(\text{CNTRL})(\text{PTD} + \mathbf{Ipac} + \Delta \mathbf{Ipac})$	$\begin{aligned} \mathbf{Nt}' &= 0.0 \\ \mathbf{Ipac}' &= 0.0 \\ \mathbf{Feac}' &= 0.0 \\ \mathbf{Ipnr}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$
IP	$\bar{t}^{IP} = \begin{cases} \emptyset & \text{if IPNR} = \emptyset \\ S(s, \text{IPCL}, T^{MD}) & \text{else} \end{cases}$ <p>with $s = \begin{cases} \emptyset & \text{if IPANX} = \emptyset \wedge \text{IPCL} = \emptyset \\ \text{IPCED} & \text{else if IPCED} \neq \emptyset \\ \text{IED} + \text{IPCL} & \text{else if IPANX} = \emptyset \\ \text{IPANX} & \text{else} \end{cases}$</p>	$X_{\text{CUR}}^{\text{CURS}}(t) \mathbf{Isc} \cdot (\mathbf{Ipac} + \Delta \mathbf{Ipac})$	$\begin{aligned} \mathbf{Ipac}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$

IPCI	$\vec{t}^{IPCI} = \begin{cases} \emptyset & \text{if } \text{IPCED} = \emptyset \\ S(s, \text{IPCL}, \text{IPCED}) & \text{else} \end{cases}$ with $s = \begin{cases} \emptyset & \text{if } \text{IPANX} = \emptyset \wedge \text{IPCL} = \emptyset \\ \text{IED} + \text{IPCL} & \text{else if } \text{IPANX} = \emptyset \\ \text{IPANX} & \text{else} \end{cases}$	$\mathbf{Nt}' = \mathbf{Nt} + \mathbf{Ipac} + \Delta\mathbf{Ipac}$ $\mathbf{Ipac}' = 0.0$ $\mathbf{Sd}' = t$
RR	$\vec{t}^{RR} = \begin{cases} \emptyset & \text{if } \text{RRANX} = \emptyset \wedge \text{RRCL} = \emptyset \\ \vec{t} \setminus t^{RRY} & \text{else if } \text{RRNXT} \neq \emptyset \\ \vec{t} & \text{else} \end{cases}$ where $\vec{t} = S(s, \text{RRCL}, T^{MD})$ with $s = \begin{cases} \text{IED} + \text{RRCL} & \text{if } \text{RRANX} = \emptyset \\ \text{RRANX} & \text{else} \end{cases}$ $t^{RRY} = \inf t \in \vec{t} \mid t > \text{SD}$	$\mathbf{Ipac}' = \mathbf{Ipac} + \Delta\mathbf{Ipac}$ $\mathbf{Ipnr}' = \min(\max(\mathbf{Ipnr} + \Delta r, \text{RRLF}), \text{RRLC})$ $\mathbf{Sd}' = t$ with $\Delta r = \min(\max(O^{rf}(\text{RRMO}, t)\text{RRMLT} + \text{RRSP} - \mathbf{Ipnr}, \text{RRPF}), \text{RRPC})$
RRF	$t^{RRF} = \begin{cases} \emptyset & \text{if } \text{RRANX} = \emptyset \wedge \text{RRCL} = \emptyset \\ \inf t \in \vec{t} \mid t > \text{SD} & \text{else} \end{cases}$ where $\vec{t} = S(s, \text{RRCL}, T^{MD})$ with $s = \begin{cases} \text{IED} + \text{RRCL} & \text{if } \text{RRANX} = \emptyset \\ \text{RRANX} & \text{else} \end{cases}$	$\mathbf{Ipac}' = \mathbf{Ipac} + \Delta\mathbf{Ipac}$ $\mathbf{Ipnr}' = \text{RRNXT}$ $\mathbf{Sd}' = t$
SC	$\vec{t}^{SC} = \begin{cases} \emptyset & \text{if } \text{SCEF} = '000'$ $S(s, \text{SCCL}, T^{MD}) & \text{else} \end{cases}$ with $s = \begin{cases} \emptyset & \text{if } \text{SCANX} = \emptyset \wedge \text{SCCL} = \emptyset \\ \text{IED} + \text{SCCL} & \text{else if } \text{SCANX} = \emptyset \\ \text{SCANX} & \text{else} \end{cases}$	$\mathbf{Ipac}' = \mathbf{Ipac} + \Delta\mathbf{Ipac}$ $\mathbf{Nsc}' = \begin{cases} \mathbf{Nsc} & \text{if } \text{SCEF} = [x]0[x] \\ \frac{O^{rf}(\text{SCMO}, t) - \text{SCIED}}{\text{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}' = \begin{cases} \mathbf{Isc} & \text{if } \text{SCEF} = 0[x][x] \\ \frac{O^{rf}(\text{SCMO}, t) - \text{SCIED}}{\text{SCIED}} & \text{else} \end{cases}$ $\mathbf{Sd}' = t$
CE	$\vec{t}^{CE} = t(e^k) \mid \mathbf{Prf}' \neq 0.0$	$\text{STF_AD_PAM}()$

Definition 12 (PAM relational step). The relational one-step transition; each constructor's target is the functional next-state **stf** of the table above, so the relation is exactly the graph of **stf** (Theorem 21).

5.2 LAM: Linear Amortizer

A linear amortizer repays principal in equal redemptions on the principal-redemption schedule PR , so the outstanding notional steps down by a fixed amount each period and reaches zero exactly at maturity. Interest is charged on the declining balance, so the interest portion of each instalment shrinks over time while the principal portion stays constant — the classic constant-amortization loan.

LAM, NAM and ANN share PAM's machinery and differ only in how the redemption is sized: LAM fixes the principal portion ($PRNXT$); NAM fixes the total payment and lets principal absorb whatever is left after interest; ANN fixes the total payment through the annuity formula. The LAM tables below therefore record only what differs from PAM — chiefly the PR redemption and the interest-calculation-base bookkeeping ($IPCB$) that tracks the balance on which interest is charged.

Definition 13 (AMORTIZERS). The amortizing contracts, defined as deltas over PAM: a linear principal redemption (LAM), interest-first redemption (NAM), and constant-instalment annuity with rate-reset re-amortization (ANN).

LAM			
State Variables			
Variable	Initialization per t_0		
Md	$Md_{t_0} = \begin{cases} MD & \text{if } MD \neq \emptyset \\ t^- + \text{ceil}(\frac{NT}{PRNXT})PRCL & \text{else} \end{cases}$ where $t^- = \begin{cases} PRANX & \text{if } PRANX \neq \emptyset \wedge PRANX \geq t_0 \\ IED + PRCL & \text{else if } IED + PRCL \geq t_0 \\ \sup t \in \tilde{t}^{PR} \mid t < t_0 & \text{else} \end{cases}$		
Nt	Same as PAM		
Ipnr	Same as PAM		
Ipac	Same as PAM		
Feac	Same as PAM		
Nsc	Same as PAM		
Isc	Same as PAM		
Prf	Same as PAM		
Sd	Same as PAM		
Prnxt	$Prnxt_{t_0} = \begin{cases} PRNXT & \text{if } PRNXT \neq \emptyset \\ NT(\text{ceil}(\frac{Y(s, T^{MD})}{Y(s, s+PRCL)}))^{-1} & \text{else} \end{cases}$ with $s = \begin{cases} PRANX & \text{if } PRANX \neq \emptyset \wedge PRANX > t_0 \\ IED + PRCL & \text{else if } PRANX = \emptyset \wedge IED + PRCL > t_0 \\ t^- & \text{else} \end{cases}$ and where $t^- = \sup t \in \tilde{t}^{PR} \mid t < t_0$		
Ipcb	$Ipcb_{t_0} = \begin{cases} 0.0 & \text{if } t_0 < IED \\ R(CNTRL)NT & \text{else if } IPCB = 'NT' \\ R(CNTRL)IPCBA & \text{else} \end{cases}$		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_PAM()

IED	Same as PAM	POF_IED_PAM()	$\mathbf{Nt}' = R(\mathbf{CNTRL})\mathbf{NT}$ $\mathbf{Ipnr}' = \mathbf{IPNR}$ $\mathbf{Ipac}' = \begin{cases} \mathbf{IPAC} & \text{if } \mathbf{IPAC} \neq \emptyset \\ y\mathbf{Nt}' \cdot \mathbf{Ipnr}' & \text{if } \mathbf{IPANX} \neq \emptyset \wedge \mathbf{IPANX} \\ 0.0 & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ $\mathbf{Ipcb}' = \begin{cases} R(\mathbf{CNTRL})\mathbf{NT} & \text{if } \mathbf{IPCB} = \text{'NT'}$ $R(\mathbf{CNTRL})\mathbf{IPCBA} & \text{else} \end{cases}$
PR	t^{PR} $S(s, \mathbf{PRCL}, T^{MD}, F)$ with $s = \begin{cases} \emptyset & \text{if } \mathbf{PRANX} = \emptyset \wedge \mathbf{PRCL} = \emptyset \\ \mathbf{IED} + \mathbf{PRCL} & \text{else if } \mathbf{PRANX} = \emptyset \\ \mathbf{PRANX} & \text{else} \end{cases}$	$X_{\text{CUR}}^{\text{CURS}}(t)R(\mathbf{CNTRL})\mathbf{Nsc}$ \mathbf{Prnxt}	$\mathbf{Nt}' = \mathbf{Nt} - R(\mathbf{CNTRL})\mathbf{Prnxt}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if } \mathbf{FEB} = \text{'NT'}$ $\frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(\mathbf{CNTRL})\mathbf{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}' = \begin{cases} \mathbf{Ipcb} & \text{if } \mathbf{IPCB} \neq \text{'NT'}$ $\mathbf{Nt}' & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
MD	$t^{MD} = \mathbf{Md}_{t_0}$	POF_MD_PAM()	$\mathbf{Nt}' = 0.0$ $\mathbf{Ipac}' = 0.0$ $\mathbf{Feac}' = 0.0$ $\mathbf{Ipcb}' = 0.0$ $\mathbf{Sd}' = t$
PP	Same as PAM	POF_PP_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if } \mathbf{FEB} = \text{'NT'}$ $\frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(\mathbf{CNTRL})\mathbf{FER} & \text{else} \end{cases}$ $\mathbf{Nt}' = \mathbf{Nt} - f(O^{ev}(\mathbf{CID}, \mathbf{PP}, t))$ $\mathbf{Ipcb}' = \begin{cases} \mathbf{Ipcb} & \text{if } \mathbf{IPCB} \neq \text{'NT'}$ $\mathbf{Nt}' & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
PY	Same as PAM	POF_PY_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if } \mathbf{FEB} = \text{'NT'}$ $\frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(\mathbf{CNTRL})\mathbf{FER} & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
FP	Same as PAM	POF_FP_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = 0.0$ $\mathbf{Sd}' = t$

PRD	Same as PAM	$X_{CUR}^{CURS}(t)R(CNTRL)(-1)(PPRD)$ $\mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if FEB} = \dots \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(CNTRL)FER & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
TD	Same as PAM	$X_{CUR}^{CURS}(t)R(CNTRL)(PTD +$ $\mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot$ \mathbf{Ipcb}	STF_TD_PAM()
IP	Same as PAM	$X_{CUR}^{CURS}(t)\mathbf{Isc} \cdot (\mathbf{Ipac} +$ $Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot$ \mathbf{Ipcb}	STF_IP_PAM()
IPCI	Same as PAM	POF_IPCI_PAM()	$\mathbf{Nt}' = \mathbf{Nt} + \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Ipac}' = 0.0$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if FEB} = \dots \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(CNTRL)FER & \text{else} \end{cases}$ $\mathbf{Ipcb}' = \begin{cases} \mathbf{Ipcb} & \text{if IPCB} \neq \text{'NT'}$ $\mathbf{Nt}' & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
IPCB	$\bar{t}^{IPCB} = 0.0$ $\begin{cases} \emptyset & \text{if IPCB} \neq \text{'NTL'}$ $S(s, IPCBCL, T^{MD}) & \text{else} \end{cases}$ with $s = \begin{cases} \emptyset & \text{if IPCBANX} = \emptyset \wedge \text{IPCBCL} = \emptyset$ $\text{IED} + \text{IPCBCL} & \text{else if IPCBANX} = \emptyset$ $\text{IPCBANX} & \text{else} \end{cases}$		$\mathbf{Ipcb}' = \mathbf{Nt}$ $\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if FEB} = \dots \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(CNTRL)FER & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
RR	Same as PAM	POF_RR_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t)\mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t)\mathbf{NtFER} & \text{if FEB} = \dots \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(CNTRL)FER & \text{else} \end{cases}$ $\mathbf{Ipnr}' = \min(\max(\mathbf{Ipnr} + \Delta r, \text{RRLF}), \text{RRLC})$ $\mathbf{Sd}' = t$ with $\Delta r = \min(\max(O^r f(\text{RRMO}, t)\text{RRMLT} + \text{RRSP} - \mathbf{Ipc}$ $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$

RRF	Same as PAM	POF_RRF_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t) \mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t) \mathbf{NtFER} & \text{if FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\mathbf{CTRL}) \mathbf{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}' = \mathbf{RRNXT}$ $\mathbf{Sd}' = t$ <p>with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$</p>
SC	Same as PAM	POF_SC_PAM()	$\mathbf{Ipac}' = \mathbf{Ipac} + Y(\mathbf{Sd}, t) \mathbf{Ipnr} \cdot \mathbf{Ipcb}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t) \mathbf{NtFER} & \text{if FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\mathbf{CTRL}) \mathbf{FER} & \text{else} \end{cases}$ $\mathbf{Nsc}' = \begin{cases} \mathbf{Nsc} & \text{if SCEF} = [x]0[x] \\ \frac{O^{rf}(\mathbf{SCM0}, t) - \mathbf{SCIED}}{\mathbf{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}' = \begin{cases} \mathbf{Isc} & \text{if SCEF} = 0[x][x] \\ \frac{O^{rf}(\mathbf{SCM0}, t) - \mathbf{SCIED}}{\mathbf{SCIED}} & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ <p>with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$</p>
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.3 LAX: Exotic Linear Amortizer

The exotic amortizer generalizes LAM to non-uniform schedules. Principal can be both drawn down (PI, principal increase) and redeemed (PR) on piecewise *array* schedules (§3): the term is partitioned into segments, each with its own anchor, cycle, rate and redemption amount. This captures structured loans with grace periods, step-ups, balloon segments and draw phases that a single cyclic schedule cannot express.

Within each segment the dynamics follow LAM; the array schedule concatenates the per-segment cyclic schedules. LAX carries the same dual model as the lending family: its own state-transition and payoff functions `LAX.stf/LAX.pof` (generalised with a per-event payload — the array redemption amount, or the reset rate), together with a relational `LAX.Step` proven to be their graph. The executable builder folds `stf/pof` over the array-derived event list, so the relation certifies the engine.

Definition 14 (LAXUMP). LAX (exotic amortizer) redeems or draws principal on piecewise array schedules; UMP (§5.7) is a non-maturity deposit whose interest capitalizes, settling at the agreed price plus accrued interest on termination. Both carry the relational `Step` and functional `stf/pof` models (proven to agree), alongside their executable builders.

LAX	
State Variables	
Variabl	Initialization per t_0

Md	$\mathbf{Md}_{t_0} = \begin{cases} \text{MD} & \text{if } \text{MD} \neq \emptyset \\ \inf t > t_0 \mid N(t) = 0 & \text{else} \end{cases}$ <p>with $N(t) = \text{NT} + \sum_{i=1}^{n(t)} (-1)^k \text{ARPRNXT}_i \mid s_i \mid$ where $n(t) = \begin{cases} \sup k \in \mathbb{N} \mid \text{ARPRANX}_k < t & \text{if } t < \max(\text{ARPRANX}) \\ \mid \text{ARPRANX} \mid & \text{else} \end{cases}$ $k = \begin{cases} 0 & \text{if } \text{ARINCDEC}_i = \text{'INC'}$ $1 & \text{else} \end{cases}$ $s_i = \begin{cases} \{\text{ARPRANX}_i\} & \text{if } \text{ARPRCL} = \emptyset \\ S(\text{ARPRANX}_i, \vec{C}_i, T_i) & \text{else} \end{cases}$ $T_i = \begin{cases} \text{ARPRANX}_{i+1} & \text{if } i < \mid \text{ARPRANX} \mid \\ t & \text{else} \end{cases}$ $\vec{C} = \begin{cases} \text{ARRRCL} & \text{if } \mid \text{ARRRCL} \mid = \mid \text{ARRRANX} \mid \\ \{c_1, c_2, \dots, c_n\} & \text{else} \end{cases}$</p>		
Nt	Same as PAM		
Ipnr	Same as PAM		
Ipac	Same as PAM		
Feac	Same as PAM		
Nsc	Same as PAM		
Isc	Same as PAM		
Prf	Same as PAM		
Sd	Same as PAM		
Prnxt	$\mathbf{Prnxt}_{t_0} = \begin{cases} 0.0 & \text{if } t_0 \geq \text{ARPRANX}_1 \\ \text{ARPRNXT}_i & \text{else} \end{cases}$ <p>where $i = \sup k \in \mathbb{N} \mid \text{ARPRANX}_k < t_0$</p>		
Ipcb	Same as LAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_PAM()
IED	Same as PAM	POF_IED_PAM()	STF_IED_LAM()
PR	$\vec{t}^{PR} = \begin{cases} \{t_1, t_2, \dots, t_i, \dots\} & \text{if } \text{ARPRCL} = \emptyset \\ s_1 \cup s_2 \cup \dots \cup s_i \cup \dots & \text{else} \end{cases}$ <p>with $s_i = S(\text{ARPRANX}_i, \vec{C}_i, \text{ARPRANX}_{i+1}),$ $i \in \{1, 2, \dots, \mid \text{ARINCDEC} \mid\}$ $\mid \text{ARINCDEC}_i = \text{'DEC'}$ </p> <p>with $\vec{C} = \begin{cases} \text{ARPRCL} & \text{if } \mid \text{ARPRCL} \mid = \mid \text{ARPRANX} \mid \\ \{c_1, c_2, \dots, c_n\} & \text{else} \end{cases}$ <p>where $n = \mid \text{ARPRANX} \mid, c_k = \text{ARRRCL}_1 \forall k$</p> </p>	POF_PR_LAM()	STF_PR_LAM()
MD	Same as PAM	POF_MD_PAM()	STF_MD_LAM()

PI	$\vec{t}^{PI} = \begin{cases} \{t_1, t_2, \dots, t_i, \dots\} & \text{if } \text{ARPRCL} = \emptyset \\ s_1 \cup s_2 \cup \dots \cup s_i \cup \dots & \text{else} \end{cases}$ with $s_i = S(\text{ARPRANX}_i, \vec{C}_i, \text{ARPRANX}_{i+1}),$ $i \in \{1, 2, \dots, \text{ARINCDEC} \}$ $\text{ARINCDEC}_i = \text{'INC'}$ with $\vec{C} = \begin{cases} \text{ARPRCL} & \text{if } \text{ARPRCL} = \text{ARPRANX} \\ \{c_1, c_2, \dots, c_n\} & \text{else} \end{cases}$ where $n = \text{ARPRANX} , c_k = \text{ARPRCL}_1 \forall k$	$X_{\text{CUR}}^{\text{CURS}}(t)R(\text{CNTRL})(-1)\text{Nsc}$ $\text{Prnxt} = \emptyset$	$\text{Nt}' = \text{Nt} + R(\text{CNTRL})\text{Prnxt}$ $\text{Ipac}' = \text{Ipac} + Y(\text{Sd}, t)\text{Ipnr} \cdot \text{Ipcb}$ $\text{Feac}' = \begin{cases} \text{Feac} + Y(\text{Sd}, t)\text{NtFER} & \text{if FEB} = \text{'FEB'}$ $\frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(\text{CNTRL})\text{FER} & \text{else} \end{cases}$ $\text{Ipcb}' = \begin{cases} \text{Ipcb} & \text{if IPCB} \neq \text{'NT'}$ $\text{Nt}' & \text{else} \end{cases}$ $\text{Sd}' = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PRF	$\vec{t}^{PRF} = \text{ARPRANX}$	0.0	$\text{Prnxt}' = \text{ARPRNXT}_i$ $\text{Ipac}' = \text{Ipac} + Y(\text{Sd}, t)\text{Ipnr} \cdot \text{Ipcb}$ $\text{Feac}' = \begin{cases} \text{Feac} + Y(\text{Sd}, t)\text{NtFER} & \text{if FEB} = \text{'FEB'}$ $\frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}R(\text{CNTRL})\text{FER} & \text{else} \end{cases}$ $\text{Ipcb}' = \begin{cases} \text{Ipcb} & \text{if IPCB} \neq \text{'NT'}$ $\text{Nt}' & \text{else} \end{cases}$ $\text{Sd}' = t$ with $i = \sup k \in \mathbb{N} \mid \text{ARPRANX}_k = t$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PP	Same as PAM	POF_PP_PAM()	STF_PP_LAM()
PY	Same as PAM	POF_PY_PAM()	STF_PY_LAM()
FP	Same as PAM	POF_FP_PAM()	STF_FP_LAM()
PRD	Same as PAM	POF_PRD_LAM()	STF_PRD_LAM()
TD	Same as PAM	POF_TD_LAM()	STF_TD_PAM()
IP	$\vec{t}^{IP} = \vec{S}(\text{ARIPANX}, \text{ARIPCL}, \text{Md}_{t_0})$	POF_IP_LAM()	STF_IP_PAM()
IPCI	Same as PAM	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	Same as LAM	POF_IPCB_LAM()	STF_IPCB_LAM()

RR	$\begin{aligned} \vec{t}^{RR} &= \text{POF_RR_PAM}() \\ \begin{cases} \{t_1, t_2, \dots, t_i, \dots\} & \text{if } \text{ARRRCL} = \emptyset \\ s_1 \cup s_2 \cup \dots \cup s_i \cup \dots & \text{else} \end{cases} \\ \text{with} \\ s_i &= \\ S(\text{ARRRANX}_i, \vec{C}_i, \text{ARRRANX}_{i+1}), \\ i \in \{1, 2, \dots, \text{ARFIXVAR} \\ \} \text{ARFIXVAR}_i = 'V' \\ \text{with} \\ \vec{C} &= \\ \begin{cases} \text{ARRRCL} & \text{if } \text{ARRRCL} = \text{ARRRANX} \\ \{c_1, c_2, \dots, c_n\} & \text{else} \end{cases} \\ \text{where} \\ n = \text{ARRRANX} , c_k &= \\ \text{ARRRCL}_1 \forall k \end{aligned}$	$\begin{aligned} \text{Ipac}' &= \text{Ipac} + Y(\text{Sd}, t) \text{Ipnr} \cdot \text{Ipcb} \\ \text{Feac}' &= \begin{cases} \text{Feac} + Y(\text{Sd}, t) \text{NtFER} & \text{if } \text{FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL}) \text{FER} & \text{else} \end{cases} \\ \text{Ipnr}' &= \min(\max(\text{Ipnr} + \Delta r, \text{RRLF}), \text{RRLC}) \\ \text{Sd}' &= t \\ \text{with} \\ \Delta r &= \min(\max(O^r f(\text{RRMO}, t) \text{RRMLT} + \text{ARRATE}_i, \\ i = \sup k \in \mathbb{N} \text{ARRRANX}_k = t \\ t^{FP-} = \sup t \in \vec{t}^{FP} t < t_0 \\ t^{FP+} = \inf t \in \vec{t}^{FP} t > t_0 \end{aligned}$	
RRF	$\begin{aligned} \vec{t}^{RRF} &= \text{POF_RRF_PAM}() \\ \begin{cases} \{t_1, t_2, \dots, t_i, \dots\} & \text{if } \text{ARRRCL} = \emptyset \\ s_1 \cup s_2 \cup \dots \cup s_i \cup \dots & \text{else} \end{cases} \\ \text{with} \\ s_i &= \\ S(\text{ARRRANX}_i, \vec{C}_i, \text{ARRRANX}_{i+1}), \\ i \in \{1, 2, \dots, \text{ARFIXVAR} \\ \} \text{ARFIXVAR}_i = 'F' \\ \text{with} \\ \vec{C} &= \\ \begin{cases} \text{ARRRCL} & \text{if } \text{ARRRCL} = \text{ARRRANX} \\ \{c_1, c_2, \dots, c_n\} & \text{else} \end{cases} \\ \text{where} \\ n = \text{ARRRANX} , c_k &= \\ \text{ARRRCL}_1 \forall k \end{aligned}$	$\begin{aligned} \text{Ipac}' &= \text{Ipac} + Y(\text{Sd}, t) \text{Ipnr} \cdot \text{Ipcb} \\ \text{Feac}' &= \begin{cases} \text{Feac} + Y(\text{Sd}, t) \text{NtFER} & \text{if } \text{FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL}) \text{FER} & \text{else} \end{cases} \\ \text{Ipnr}' &= \text{ARRATE}_i \\ \text{Sd}' &= t \\ \text{with} \\ i = \sup k \in \mathbb{N} \text{ARRRANX}_k = t \\ t^{FP-} = \sup t \in \vec{t}^{FP} t < t_0 \\ t^{FP+} = \inf t \in \vec{t}^{FP} t > t_0 \end{aligned}$	
SC	Same as PAM	POF_SC_PAM()	STF_SC_LAM()
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.4 NAM: Negative Amortizer

In a negative amortizer the borrower pays a fixed total instalment (**PRNXT**) each period, out of which interest is served first and only the remainder reduces principal. When the instalment exceeds the interest due the loan amortizes normally; when interest exceeds the instalment the shortfall is capitalized and the outstanding notional actually *grows* — hence “negative” amortization.

NAM is PAM with a PR event whose redemption equals the fixed payment net of accrued interest, with the interest-calculation base updated accordingly. It is the model for payment-capped and graduated-payment mortgages, where the scheduled payment is held constant even when it fails to cover current interest.

NAM			
State Variables			
Variabl	Initialization per t_0		
Md	$\mathbf{Md}_{t_0} = \begin{cases} \text{MD} & \text{if } \text{MD} \neq \emptyset \\ t^- + n\text{PRCL} & \text{else} \end{cases}$ with $n = \text{ceil}(\frac{\text{NT}}{\text{PRNXT} - \text{NTY}(t^-, t^- + \text{PRCL}) \text{IPNR}})$ where $t^- = \begin{cases} \text{PRANX} & \text{if } \text{PRANX} \neq \emptyset \wedge \text{PRANX} \geq t_0 \\ \text{IED} + \text{PRCL} & \text{else if } \text{IED} + \text{PRCL} \geq t_0 \\ \sup t \in t^{PR} \mid t < t_0 & \text{else} \end{cases}$		
Nt	Same as PAM		
Ipnr	Same as PAM		
Ipac	Same as PAM		
Feac	Same as PAM		
Nsc	Same as PAM		
Isc	Same as PAM		
Prf	Same as PAM		
Sd	Same as PAM		
Prnxt	$\mathbf{Prnxt}_{t_0} = R(\text{CNTRL})\text{PRNXT}$		
Ipcb	Same as LAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	$\text{POF_AD_PAM}()$	$\text{STF_AD_PAM}()$
IED	Same as PAM	$\text{POF_IED_PAM}()$	$\text{STF_IED_LAM}()$
PR	Same as LAM	$X_{\text{CUR}}^{\text{CURS}}(t) \text{Nsc} \cdot (\mathbf{Prnxt} - \mathbf{Ipac} - Y(\text{Sd}, t) \text{Ipnr} \cdot \mathbf{Ipcb})$	$\begin{aligned} \mathbf{Nt}' &= \mathbf{Nt} - (\mathbf{Prnxt} - \mathbf{Ipac}') \\ \mathbf{Ipac}' &= \mathbf{Ipac} + Y(\text{Sd}, t) \text{Ipnr} \cdot \mathbf{Ipcb} \\ \mathbf{Feac}' &= \begin{cases} \mathbf{Feac} + Y(\text{Sd}, t) \text{Nt} \text{FER} & \text{if } \text{FEB} = \text{'NT'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL}) \text{FER} & \text{else} \end{cases} \\ \mathbf{Ipcb}' &= \begin{cases} \mathbf{Ipcb} & \text{if } \text{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}' & \text{else} \end{cases} \\ \mathbf{Sd}' &= t \end{aligned}$ with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$
MD	Same as PAM	$\text{POF_MD_PAM}()$	$\text{STF_MD_LAM}()$
PP	Same as PAM	$\text{POF_PP_PAM}()$	$\text{STF_PP_LAM}()$
PY	Same as PAM	$\text{POF_PY_PAM}()$	$\text{STF_PY_LAM}()$
FP	Same as PAM	$\text{POF_FP_PAM}()$	$\text{STF_FP_LAM}()$
PRD	Same as PAM	$\text{POF_PRD_LAM}()$	$\text{STF_PRD_LAM}()$
TD	Same as PAM	$\text{POF_TD_LAM}()$	$\text{STF_TD_PAM}()$

IP	$\vec{t}^{IP} = (\vec{u}, \vec{v})$ where $\vec{u} = \begin{cases} \emptyset & \text{if } IPANX = \emptyset \wedge IPCL = \emptyset \\ \emptyset & \text{if } IPCED \neq \emptyset \wedge IPCED \geq T \\ S(r, IPCL, T) & \text{else} \end{cases}$ $\vec{v} = S(s, PRCL, T^{MD})$ with $r = \begin{cases} IPCED & \text{if } IPCED \neq \emptyset \\ IPANX & \text{else if } IPANX \neq \emptyset \\ IED + IPCL & \text{else if } IPCL \neq \emptyset \\ \emptyset & \text{else} \end{cases}$ $T = s - PRCL$ $s = \begin{cases} IED + PRCL & \text{if } PRANX = \emptyset \\ PRANX & \text{else} \end{cases}$	POF_IP_LAM()	STF_IP_PAM()
IPCI	Same as PAM	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	Same as LAM	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	Same as PAM	POF_RR_PAM()	STF_RR_LAM()
RRF	Same as PAM	POF_RRF_PAM()	STF_RRF_LAM()
SC	Same as PAM	POF_SC_PAM()	STF_SC_LAM()
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.5 ANN: Annuity

An annuity pays a constant total instalment over its life — the annuity amount A of §3 — sized so that the level payments exactly retire the loan by maturity. Early instalments are mostly interest and later ones mostly principal, but the total is fixed, which is what makes annuities the standard structure for consumer credit and mortgage lending.

Because the instalment depends on the prevailing rate, a rate reset (RR) triggers a re-amortization: the annuity amount is recomputed from the current balance, the new rate and the remaining redemption schedule so the contract still amortizes to zero by MD. Apart from this re-sizing, ANN reuses PAM’s rate, interest and fee logic — which is why the rate-bound metatheorem (24) carries over to ANN for free.

ANN	
State Variables	
Variabl	Initialization per t_0
Md	Same as NAM
Nt	Same as PAM
Ipnr	Same as PAM
Ipac	Same as PAM
Feac	Same as PAM
Nsc	Same as PAM
Isc	Same as PAM
Prf	Same as PAM
Sd	Same as PAM

Prnxt	$\text{Prnxt}_{t_0} = \begin{cases} R(\text{CNTRL})\text{PRNXT} & \text{if } \text{PRNXT} \neq \emptyset \\ (\text{NT} + \text{Ipac}_{t_0}) \frac{t_{\text{odo}}}{t_0} & \text{else} \end{cases}$ where $n = \vec{t} $ with $ a $ indicating the cardinality of set a		
Ipcb	Same as LAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_PAM()
IED	Same as PAM	POF_IED_PAM()	STF_IED_LAM()
PR	Same as LAM	POF_PR_NAM()	STF_PR_NAM()
MD	Same as PAM	POF_MD_PAM()	STF_MD_LAM()
PP	Same as PAM	POF_PP_PAM()	STF_PP_LAM()
PY	Same as PAM	POF_PY_PAM()	STF_PY_LAM()
FP	Same as PAM	POF_FP_PAM()	STF_FP_LAM()
PRD	Same as PAM	POF_PRD_LAM()	STF_PRD_LAM()
TD	Same as PAM	POF_TD_LAM()	STF_TD_PAM()
IP	Same as NAM	POF_IP_LAM()	STF_IP_PAM()
IPCI	Same as PAM	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	Same as LAM	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	Same as PAM	POF_RR_PAM()	$\text{Ipac}' = \text{Ipac} + Y(\text{Sd}, t)\text{Ipnr} \cdot \text{Ipcb}$ $\text{Feac}' = \begin{cases} \text{Feac} + Y(\text{Sd}, t)\text{NtFER} & \text{if } \text{FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL})\text{FER} & \text{else} \end{cases}$ $\text{Ipnr}' = \min(\max(\text{Ipnr} + \Delta r, \text{RRLF}), \text{RRLC})$ $\text{Prnxt}' = A(t, \text{Md}', \text{Nt}', \text{Ipac}', \text{Ipnr}')$ $\text{Sd}' = t$ <p>with</p> $\Delta r = \min(\max(O^{rf}(\text{RRMO}, t)\text{RRMLT} + \text{RRSP} - \text{Ip}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	Same as PAM	POF_RRF_PAM()	$\text{Ipac}' = \text{Ipac} + Y(\text{Sd}, t)\text{Ipnr} \cdot \text{Ipcb}$ $\text{Feac}' = \begin{cases} \text{Feac} + Y(\text{Sd}, t)\text{NtFER} & \text{if } \text{FEB} = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL})\text{FER} & \text{else} \end{cases}$ $\text{Ipnr}' = \text{RRNXT}$ $\text{Prnxt}' = A(t, \text{Md}', \text{Nt}', \text{Ipac}', \text{Ipnr}')$ $\text{Sd}' = t$ <p>with</p> $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	Same as PAM	POF_SC_PAM()	STF_SC_LAM()
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.6 CLM: Call Money

Call money is a short-term, open loan that either party may “call” (terminate) at will, subject to a notice period. It has no contractual maturity: interest is not paid out but capitalizes into the notional through IPCI, so the balance compounds until the loan is called.

Termination is driven by an observed exercise event ($\text{XD} = \sup \tau(O^{ev}(\text{CID},))$); after the notice

period the grown notional is settled in a single payment (STD). Apart from the open maturity and the capitalization of interest, CLM follows PAM, so it reuses PAM's state transitions directly.

Definition 15 (CLM). Call money: PAM state transitions with interest capitalizing until maturity. The open “call has no fixed maturity — it closes on an observed exercise ($\mathbf{XD} = \sup \tau(O^{ev}(\mathbf{CID},))$), settling the grown notional after the exercise-notice period.

CLM			
State Variables			
Variable	Initialization per t_0		
Md	$\mathbf{Md}_{t_0} = \begin{cases} \mathbf{MD} & \text{if } \mathbf{MD} \neq \emptyset \\ s & \text{else if } O^{ev}(\mathbf{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$ where $s = \sup t \in \tau(O^{ev}(\mathbf{CID}, t_0))$		
Nt	Same as PAM		
Ipnr	Same as PAM		
Ipac	Same as PAM		
Feac	Same as PAM		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_PAM()
IED	Same as PAM	$X_{CUR}^{CURS}(t)R(\mathbf{CNTRL})(-1)\mathbf{NT}$	STF_IED_PAM()
PR	Same as PAM	POF_PR_PAM()	STF_PR_PAM()
FP	Same as PAM	POF_FP_PAM()	STF_FP_PAM()
IP	$t^{IP} = \mathbf{Md}_{t_0}$	$X_{CUR}^{CURS}(t)(\mathbf{Ipac} + \Delta\mathbf{Ipac})$	$\mathbf{Ipac}' = 0.0$ $\mathbf{Sd}' = t$
IPCI	$\begin{cases} t^{IPCI} & = \\ \emptyset & \text{if } \mathbf{IPNR} = \emptyset \\ S(s, \mathbf{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$ where $\begin{cases} s & = \\ \mathbf{IPANX} & \text{if } \mathbf{IPANX} \neq \emptyset \\ \mathbf{IED} + \mathbf{IPCL} & \text{else} \end{cases}$	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	Same as PAM	POF_RR_PAM()	STF_RR_PAM()
RRF	Same as PAM	POF_RRF_PAM()	STF_RRF_PAM()
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.7 UMP: Undefined Maturity Profile

The undefined-maturity profile models non-maturity deposits such as savings and current accounts: a principal balance that may change over time, interest that capitalizes rather than being paid out, and no scheduled maturity. The holder may add to or withdraw from the balance, and on termination the account settles at its agreed value plus accrued interest.

Like CLM, UMP has no **MD**; interest accrues on the running balance and is capitalized into it (**IPCI**), the deposit settling at **PTD** plus the accrued interest on termination. UMP carries the

dual relational/functional model (`UMP.Step`, `UMP.stf/pof`); it is documented jointly with LAX as 14.

UMP			
State Variables			
Variabl	Initialization per t_0		
Md	$\mathbf{Md}_{t_0} = \begin{cases} s & \text{if } O^{ev}(\mathbf{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$ where $s = \sup t, t \in \tau(O^{ev}(\mathbf{CID}, i, t_0))$ $i \in \{\mathbf{PR}, \mathbf{PI}\}$		
Nt	Same as PAM		
Ipnr	Same as PAM		
Ipac	Same as PAM		
Feac	Same as PAM		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_PAM()
IED	Same as PAM	POF_IED_CLM()	STF_IED_PAM()
PR	$\bar{t}^{PR} = O^{ev}(\mathbf{CID}, i, t_0)$ with $i \in \{\mathbf{PR}, \mathbf{PI}\}$	$f(e_t^{PR})$	$\begin{aligned} \mathbf{Ipac}' &= \mathbf{Ipac} + \Delta\mathbf{Ipac} \\ \mathbf{Feac}' &= \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t) & \text{if } \mathbf{FEB} = \mathbf{N} \\ \frac{Y(t^-, t)}{Y(t^-, t^+)} \mathbf{FER} & \text{else} \end{cases} \\ \mathbf{Nt}' &= \mathbf{Nt} - f(e_t^{PR}) \\ \mathbf{Sd}' &= t \end{aligned}$
FP	Same as PAM	POF_FP_PAM()	STF_FP_PAM()
IPCI	$\bar{t}^{IPCI} = \begin{cases} \emptyset & \text{if } \mathbf{IPNR} = \emptyset \\ S(s, \mathbf{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$ where $s = \begin{cases} \mathbf{IPANX} & \text{if } \mathbf{IPANX} \neq \emptyset \\ \mathbf{IED} + \mathbf{IPCL} & \text{else} \end{cases}$	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	Same as PAM	POF_RR_PAM()	STF_RR_PAM()
RRF	Same as PAM	POF_RRF_PAM()	STF_RRF_PAM()
CE	Same as PAM	POF_CE_PAM()	STF_AD_PAM()

5.8 CSH: Cash

Cash is the simplest contract: a pure holding of money, with no schedule, no interest and no maturity. Its only state is the notional, and its value at any time is just that balance carried with the contract-role sign. CSH exists so that cash positions can sit alongside other instruments in a portfolio and be analyzed uniformly; it generates no scheduled cash flow of its own, which is why its tables below are essentially empty.

CSH			
State Variables			

COM			
State Variables			
Variabl	Initialization per t_0		
Sd	Same as STK		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_STK()
PRD	Same as STK	POF_PRD_STK()	STF_PRD_STK()
TD	Same as STK	POF_PRD_STK()	STF_PRD_STK()

5.11 FXOUT: Foreign Exchange Outright

An FX outright is an agreement to exchange two fixed notionals in different currencies on a single future settlement date at a pre-agreed rate. One leg delivers NT units of the base currency and the other delivers the counter-currency amount; settlement may be physical (both legs exchanged) or in net cash.

The payoff is therefore the signed difference of the two delivery legs, converted at the settlement-currency rate. The conservation metatheorem (36) records that the two legs net exactly to the par difference $R \cdot (NT_1 - NT_2)$ at unit FX. FXOUT is one of the cash-settled derivatives formalized as 17.

Definition 17 (DERIVATIVE). Cash-settled derivatives: a European option's intrinsic value (OPTNS, §5.15), a future's linear payoff $S - F$ (FUTUR, §5.16), an FX outright's two-currency settlement (FXOUT), and an interest-rate cap/floors rate-excess over a child contract's reset rate (CAPFL, §5.14).

FXOUT			
State Variables			
Variabl	Initialization per t_0		
Md	$Md_{t_0} = \begin{cases} MD & \text{if } STD = \emptyset \\ STD & \text{else} \end{cases}$		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	STF_AD_STK()
PRD	Same as PAM	POF_PRD_STK()	STF_PRD_STK()
TD	Same as PAM	POF_TD_STK()	STF_TD_STK()
STD	$t^{STD} = \begin{cases} \emptyset & \text{if } DS = 'D' \\ Md_{t_0} & \text{else} \end{cases}$	$X_{CUR}^{CURS}(t)R(CNTRL)(NT - O^{r_f}(i, Md_t)NT_2)$ where $i = concat(CUR2, "/", CUR)$ and $concat(x, y, z)$ indicates the string concatenation function	$Sd' = t$

STD ⁽¹⁾	t^{STD} $\begin{cases} \emptyset & \text{if DS = 'S'} \\ \mathbf{Md}_{t_0} & \text{else} \end{cases}$	=	$X_{\text{CUR}}^{\text{CURS}}(t)R(\text{CNTRL})\text{NT}$	$\mathbf{Sd}' = t$
STD ⁽²⁾	t^{STD} $\begin{cases} \emptyset & \text{if DS = 'S'} \\ \mathbf{Md}_{t_0} & \text{else} \end{cases}$	=	$X_{\text{CUR}}^{\text{CURS}}(t)R(\text{CNTRL})(-1)\text{NT}2$	$\mathbf{Sd}' = t$
CE	Same as PAM		$\text{POF_CE_PAM}()$	$\text{STF_CD_STK}()$

5.12 SWPPV: Plain Vanilla Interest Rate Swap

A plain-vanilla interest-rate swap exchanges a fixed-rate and a floating-rate interest stream on a common notional over a sequence of periods. On each payment date the fixed leg pays $N \cdot \text{fix} \cdot Y$ and the floating leg pays $N \cdot \text{flt} \cdot Y$ for the period year-fraction Y ; the contract may settle the two legs gross (both flows) or net (their difference).

The net settlement is exactly $R \cdot N \cdot (\text{fix} - \text{flt}) \cdot Y$, which the formalization proves equals the difference of the two gross legs (36). SWPPV is given the dual model (SWPPV.Step , SWPPV.stf/pof): a period emits the two legs IPFX/IPFL (the floating leg advances the period boundary \mathbf{Sd} ; a rate reset RR only repricings the floating rate), and the relation is proven to be the graph of stf . SWPPV and the composite SWAPS are formalized as 18.

Definition 18 (SWAP). SWPPV pays a fixed and a floating interest leg per period (with the relational Step and functional stf/pof models, proven to agree); SWAPS (§5.13) is a composite of two child legs run by the ordinary engine and combined gross or net.

SWPPV			
State Variables			
Variabl	Initialization per t_0		
\mathbf{Md}	Same as PAM		
\mathbf{Nt}	Same as PAM		
\mathbf{Ipnr}	$\mathbf{Ipnr}_{t_0} = \begin{cases} 0.0 & \text{if IED} > t_0 \\ \text{IPNR2} & \text{else} \end{cases}$		
\mathbf{Ipac}	$\mathbf{Ipac}_{t_0} = \begin{cases} \text{IPAC} & \text{if IPAC} \neq \emptyset \\ Y(t^-, t_0)\mathbf{Nt}_{t_0}(\text{IPNR} - \mathbf{Ipnr}_{t_0}) & \text{else} \end{cases}$ with $t^- = \sup t, t \in t^{IP}, t < t_0$		
$\mathbf{Ipac1}$	$\mathbf{Ipac1}_{t_0} = Y(t^-, t_0)\mathbf{Nt}_{t_0} \text{IPNR}$ with $t^- = \sup t, t \in t^{IP}, t < t_0$		
$\mathbf{Ipac2}$	$\mathbf{Ipac2}_{t_0} = Y(t^-, t_0)\mathbf{Nt}_{t_0} \mathbf{Ipnr}_{t_0}$ with $t^- = \sup t, t \in t^{IP}, t < t_0$		
\mathbf{Prf}	Same as PAM		
\mathbf{Sd}	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	$\text{POF_AD_PAM}()$	$\begin{aligned} \mathbf{Ipac}' &= Y(\mathbf{Sd}, t)\mathbf{Nt}_{t_0}(\text{IPNR} - \mathbf{Ipnr}_{t_0}) \\ \mathbf{Ipac1}' &= Y(\mathbf{Sd}, t)\mathbf{Nt}_{t_0} \text{IPNR} \\ \mathbf{Ipac2}' &= Y(\mathbf{Sd}, t)\mathbf{Nt}_{t_0} \mathbf{Ipnr}_{t_0} \\ \mathbf{Sd}' &= t \end{aligned}$

PRD	Same as PAM	POF_PRD_STK()	$\begin{aligned} \mathbf{Ipac}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} (\mathbf{IPNR} - \mathbf{Ipnr}_{t_0}) \\ \mathbf{Ipac1}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} \mathbf{IPNR} \\ \mathbf{Ipac2}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} \mathbf{Ipnr}_{t_0} \\ \mathbf{Sd}' &= t \end{aligned}$
TD	Same as PAM	POF_TD_STK()	$\begin{aligned} \mathbf{Nt}' &= 0.0 \\ \mathbf{Ipac}' &= 0.0 \\ \mathbf{Ipac1}' &= 0.0 \\ \mathbf{Ipac2}' &= 0.0 \\ \mathbf{Ipnr}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$
IED	Same as PAM	0.0	$\begin{aligned} \mathbf{Nt}' &= R(\mathbf{CNTRL}) \mathbf{NT} \\ \mathbf{Ipac}' &= 0.0 \\ \mathbf{Ipac1}' &= 0.0 \\ \mathbf{Ipac2}' &= 0.0 \\ \mathbf{Ipnr}' &= \mathbf{IPNR2} \\ \mathbf{Sd}' &= t \end{aligned}$
PR	Same as PAM	0.0	$\begin{aligned} \mathbf{Nt}' &= 0.0 \\ \mathbf{Ipnr}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$
IP	$\begin{aligned} \bar{t}^{IP} &= \begin{cases} \emptyset & \text{if } \mathbf{DS} = \mathbf{X}_{\text{CUR}}^{\text{CURS}}(t) R(\mathbf{CNTRL})(\mathbf{Ipac} + \mathbf{Ipnr} - \mathbf{Nt}) \\ \mathbf{Md}_{t_0} & \text{else if } \mathbf{IPCL} = \emptyset \\ S(s, \mathbf{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases} \\ \text{where} & \\ s &= \begin{cases} \mathbf{IPANX} & \text{if } \mathbf{IPANX} \neq \emptyset \\ \mathbf{IED} + \mathbf{IPCL} & \text{else} \end{cases} \end{aligned}$	$\begin{aligned} \mathbf{Ipac}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$	
IP ^(fix)	$\bar{t}^{IP^{(fix)}} = \begin{cases} \emptyset & \text{if } \mathbf{DS} = \mathbf{X}_{\text{CUR}}^{\text{CURS}}(t) R(\mathbf{CNTRL})(\mathbf{Ipac1} + \mathbf{Ipnr} - \mathbf{Nt}) \\ \mathbf{Md}_{t_0} & \text{else if } \mathbf{IPCL} = \emptyset \\ S(s, \mathbf{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	$\begin{aligned} \mathbf{Ipac1}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$	
IP ^(var)	$\bar{t}^{IP^{(var)}} = \begin{cases} \emptyset & \text{if } \mathbf{DS} = \mathbf{X}_{\text{CUR}}^{\text{CURS}}(t) R(\mathbf{CNTRL})(\mathbf{Ipac2} - \Delta \mathbf{Ipac}) \\ \mathbf{Md}_{t_0} & \text{else if } \mathbf{IPCL} = \emptyset \\ S(s, \mathbf{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	$\begin{aligned} \mathbf{Ipac2}' &= 0.0 \\ \mathbf{Sd}' &= t \end{aligned}$	
RR	$\bar{t}^{RR} = S(s, \mathbf{RRCL}, \mathbf{Md}_{t_0})$ <p>where</p> $s = \begin{cases} \mathbf{RRANX} & \text{if } \mathbf{RRANX} \neq \emptyset \\ \mathbf{IED} + \mathbf{RRCL} & \text{else} \end{cases}$	POF_RR_PAM()	$\begin{aligned} \mathbf{Ipac}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} (\mathbf{IPNR} - \mathbf{Ipnr}_{t_0}) \\ \mathbf{Ipac1}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} \mathbf{IPNR} \\ \mathbf{Ipac2}' &= Y(\mathbf{Sd}, t) \mathbf{Nt}_{t_0} \mathbf{Ipnr}_{t_0} \\ \mathbf{Ipnr}' &= \mathbf{RRMLT}^{Orf}(\mathbf{RRMO}, t) + \mathbf{RRSP} \\ \mathbf{Sd}' &= t \end{aligned}$
CE	Same as PAM	POF_CE_PAM()	STF_AD_SWPPV()

5.13 SWAPS: Swap

The general swap is a *composite* contract: it references two child contracts — its two legs — runs each through the ordinary execution engine, and then combines their cash flows. The legs may be any ACTUS contracts (for instance a fixed-rate and a floating-rate PAM or ANN), and the swap settles them either gross (passing both legs' flows through) or net (offsetting same-date flows).

This compositional definition is what lets ACTUS express swaptions, asset swaps and other structured exchanges as hierarchies of simpler contracts (§4). SWAPS is formalized with SWPPV as 18.

SWAPS			
State Variables			
Variabl	Initialization per t_0		
Md	$\text{Md}_{t_0} = \max(U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t_0, \mathbf{Tmd}), U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t_0, \mathbf{Tmd}))$		
Ipac	$\text{Ipac}_{t_0} = U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t_0, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) + U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t_0, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(2)}\})$ <p>with</p> $r^{(1)} = \begin{cases} RPA & \text{if CNTRL} = RFL \\ RPL & \text{else} \end{cases}$ $r^{(2)} = \begin{cases} RPL & \text{if CNTRL} = RFL \\ RPA & \text{else} \end{cases}$		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	$\text{Ipac}' = U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) + U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(2)}\})$ $\text{Feac}' = \begin{cases} \text{Feac} + Y(\text{Sd}, t) & \text{if FEB} = \text{'N'} \\ \frac{Y(t^-, t)}{Y(t^-, t^+)} \text{FER} & \text{else} \end{cases}$ $\text{Sd}' = t$
PRD	Same as PAM	$X_{CUR}^{CURS}(t)((-1)\text{PPRD} + U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) + U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(2)}\}))$	$\text{Ipac}' = U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) + U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(2)}\})$ $\text{Sd}' = t$
TD	Same as PAM	$X_{CUR}^{CURS}(t)(\text{PTD} + U^{sv}(\text{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) + U^{sv}(\text{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(2)}\}))$	$\text{Ipac}' = 0.0$ $\text{Sd}' = t$

k	$\begin{aligned} & \{e_t^k\} = \\ & \begin{cases} \{e_t^{k,1}\} \cup \{e_s^{l,2}\} & \text{if } \text{DS} = \text{'D'} \\ \{e_t^{k,1}\} + \{e_s^{l,2}\} & \text{else} \end{cases} \\ & \text{with} \\ & \{e_t^{k,1}\} + \{e_s^{l,2}\} = U \cup V \\ & \text{and} \\ & U = \{e_t^{k,1}\} \triangle \{e_s^{l,2}\} \\ & V = \{x_\tau^m + y_\tau^m\} \\ & \text{where for any two events} \\ & x_t^k \in \{e_t^{k,1}\}, y_s^l \in \{e_s^{l,2}\} \\ & \text{we have } x_t^k = y_s^l \iff t = \\ & s \wedge k = l, \triangle \text{ is the } \textit{dis-} \\ & \textit{tinct union-operator}, \text{ and} \\ & x_t^k + y_s^l = z_\tau^m \text{ with } \tau = \\ & t = s, m = k = \\ & l \in \{\text{IED, IP, PR}\} \text{ indi-} \\ & \text{cates that any two con-} \\ & \text{gruent events of type IED,} \\ & \text{IP, or PR are } \textit{merged} \text{ into} \\ & \text{a new } \textit{aggregate} \text{ event (see} \\ & \text{payoff and state transition} \\ & \text{function below).} \\ & \text{with} \\ & \{e_t^{k,1}\} = U^{ev}(\text{CTST}_{\text{FirstLeg}}^{\text{Contract}}, t_0 \mid \{\text{CNTRL} = r^{(1)}\}) \\ & \{e_s^{l,2}\} = U^{ev}(\text{CTST}_{\text{SecondLeg}}^{\text{Contract}}, t_0 \mid \{\text{CNTRL} = r^{(2)}\}) \\ & r^{(1)} = \begin{cases} \text{RPA} & \text{if CNTRL} = \text{RFL} \\ \text{RPL} & \text{else} \end{cases} \\ & r^{(2)} = \begin{cases} \text{RPL} & \text{if CNTRL} = \text{RFL} \\ \text{RPA} & \text{else} \end{cases} \end{aligned}$		
CE	Same as PAM	POF_CE_PAM()	STF_AD_SWAPS()
z_τ^m		$f(x_\tau^m) + f(y_\tau^m)$	$\text{Ipac}' = U^{sv}(\text{CTST}_{\text{FirstLeg}}^{\text{Contract}}, t, \text{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) \\ + U^{sv}(\text{SecondLeg}, t, \text{Ipac} \mid \{\text{CNTRL} = r^{(2)}\}) \\ \text{Sd}' = t$

5.14 CAPFL: Cap–Floor

A cap (floor) is a rate option layered on a variable-rate underlying: it pays the holder whenever the underlying's reset rate rises above the cap (falls below the floor), compensating the rate excess on the underlying's notional. It is the standard hedge against adverse rate moves on a floating-rate loan.

CAPFL is defined over a child contract: at each of the underlying's reset dates it observes the would-be reset rate r and pays $\max(r - \text{cap}, 0)$ or $\max(\text{floor} - r, 0)$ on the period's notional and year-fraction. The intrinsic-value metatheorem (35) proves these payoffs are always non-negative. Formalized with the derivatives as 17.

CAPFL	
State Variables	
Variabl	Initialization per t_0

Md	$\mathbf{Md}_{t_0} = \max(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t_0, \mathbf{Tmd} \mid \{\text{CNTRL} = RPA\}), U^{sv}(\text{CTST}_{Underlying}^{Contract}, t_0, \mathbf{Tmd} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\}))$		
Ipac	$\mathbf{Ipac}_{t_0} = R(\text{CNTRL})\text{abs}(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t_0, \mathbf{Ipac} \mid \{\text{CNTRL} = r^{(1)}\}) - U^{sv}(\text{CTST}_{Underlying}^{Contract}, t_0, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\}))$		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	$\mathbf{Ipac}' = R(\text{CNTRL})\text{abs}(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA\}) - U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\}))$ $\mathbf{Sd}' = t$
PRD	Same as PAM	$X_{CUR}^{CURS}(t)((-1)\text{PPRD} + R(\text{CNTRL})\text{abs}(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA\}) - U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\})))$	$\mathbf{Ipac}' = R(\text{CNTRL})\text{abs}(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA\}) - U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\}))$ $\mathbf{Sd}' = t$
TD	Same as PAM	$X_{CUR}^{CURS}(t)(\text{PTD} + R(\text{CNTRL})\text{abs}(U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA\}) - U^{sv}(\text{CTST}_{Underlying}^{Contract}, t, \mathbf{Ipac} \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\})))$	$\mathbf{Ipac}' = 0.0$ $\mathbf{Sd}' = t$

k	$\{e_t^k\} = \{x_t^k + y_s^l\}$ for all events $x_t^k \in U^{ev}(\text{CTST}_{Underlying}^{Contract}, t_0 \mid \{\text{CNTRL} = RPA\})$, $y_s^l \in U^{ev}(\text{CTST}_{Underlying}^{Contract}, t_0 \mid \{\text{CNTRL} = RPA, \text{RRLC} = \text{RRLC}, \text{RRLF} = \text{RRLF}\})$ with $t = s \wedge k = l = IP$. That is, any two congruent events of the child-contract schedule, evaluated once without RRLC, RRLF defined and once with the attributes defined, which are of type IP are <i>merged</i> into a new <i>aggregate</i> event (see payoff and state transition function below).		
CE	Same as PAM	POF_CE_PAM()	STF_AD_CAPFL()
z_τ^m		$R(\text{CNTRL})abs(f(x_\tau^m) - f(y_\tau^m))$ where $abs(u)$ defines that the absolute value of u is taken.	Ipac' = 0.0 Sd' = t

5.15 OPTNS: Option

An option is a European call or put on an underlying instrument, exercised at a single maturity. At exercise it cash-settles at its intrinsic value — $\max(S - K, 0)$ for a call, $\max(K - S, 0)$ for a put — where S is the observed underlying price and K the strike; before maturity it carries only its premium.

Like CAPFL it is defined over a child contract through the child-contract observer (§4). The formalization proves the intrinsic value is non-negative and that a call is in the money exactly when $S > K$ (35). Formalized with the derivatives as 17.

OPTNS			
State Variables			
Variabl	Initialization per t_0		
Xd	$\mathbf{Xd}_{t_0} = \text{XD}$		
Xa	$\mathbf{Xa}_{t_0} = \text{XA}$		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	Sd' = t
PRD	Same as PAM	$X_{\text{CUR}}^{\text{CURS}}(t)(-1)\text{PPRD}$	STF_PRD_STK()

TD	Same as PAM	$X_{CUR}^{CURS}(t)PTD$	$STF_TD_STK()$
MD	$t^{MD} = MD$	0.0	$\mathbf{Xd}' = \begin{cases} \mathbf{Xd} & \text{if } \mathbf{Xd} \neq \emptyset \\ \emptyset & \text{else if } x = 0.0 \\ t & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $x = \begin{cases} \max(S_t - OPS1, 0) & \text{if } OPTP = 'C' \\ \max(OPS1 - S_t, 0) & \text{else if } OPTP = 'P' \\ \max(S_t - OPS1, 0) & \text{else} \\ + \max(OPS2 - S_t, 0) \end{cases}$ $S_t = Orf(U^{ca}(\mathbf{CTST}_{Underlying}^{Contract}, MOC), t)$
XD	$t^{XD} = \begin{cases} \mathbf{Xd}_{t_0} & \text{if } \mathbf{Xd}_{t_0} \neq \emptyset \\ \inf \tau(e^{XD}) & \text{else if } \tau(e^{XD}) \neq \emptyset \wedge OPXT \neq 'E' \\ \emptyset & \text{else} \end{cases}$ with $e^{XD} = e \in O^{ev}(i, XD, t_0) \tau(e) \leq \mathbf{Md}_{t_0}$ and $i = \mathbf{CTST}_{CoveredContract}^{Contract}(i)$	0.0	$\mathbf{Xa}' = \begin{cases} \max(S_t - OPS1, 0) & \text{if } OPTP = 'C' \\ \max(OPS1 - S_t, 0) & \text{else if } OPTP = 'P' \\ \max(S_t - OPS1, 0) & \text{else} \\ + \max(OPS2 - S_t, 0) \end{cases}$ $\mathbf{Sd}' = t$ with $S_t = Orf(U^{ca}(\mathbf{CTST}_{Underlying}^{Contract}, MOC), t)$
STD	$t^{STD} = \begin{cases} \emptyset & \text{if } XD = \emptyset \\ XD + STD & \text{else} \end{cases}$	$X_{CUR}^{CURS}(t)R(\mathbf{CNTRL})\mathbf{Xa}$	$\mathbf{Xa}' = 0.0$ $\mathbf{Sd}' = t$
CE	Same as PAM	$POF_CE_PAM()$	$STF_AD_OPTNS()$

5.16 FUTUR: Future

A future is a linear forward on an underlying: at settlement it pays the difference between the observed underlying price S and the agreed contract (futures) price F , that is $S - F$, scaled by the contract size. Unlike an option it is symmetric — the holder gains or loses linearly with the underlying — so its payoff may take either sign.

FUTUR is the simplest of the derivative builders defined over a child underlying, and is formalized with them as 17.

FUTUR			
State Variables			
Variabl	Initialization per t_0		
Xd	Same as OPTNS		
Xa	Same as OPTNS		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	$POF_AD_PAM()$	$STF_AD_OPTNS()$

PRD	Same as PAM	POF_PRD_OPTNS()	STF_PRD_STK()
TD	Same as PAM	POF_TD_OPTNS()	STF_TD_STK()
MD	Same as OPTNS	0.0	$\mathbf{Xd}' = \begin{cases} \emptyset & \text{if } x = 0.0 \\ t & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $x = S_t - \text{PFUT}$ $S_t = \text{Or}^f(U^{ca}(\text{CTST}_{\text{Underlying}}^{\text{Contract}}, \text{MOC}), t)$
XD	Same as OPTNS	0.0	$\mathbf{Xa}' = S_t - \text{PFUT}$ $\mathbf{Sd}' = t$ with $S_t = \text{Or}^f(U^{ca}(\text{CTST}_{\text{Underlying}}^{\text{Contract}}, \text{MOC}), t)$
STD	Same as OPTNS	POF_STD_OPTNS()	STF_STD_OPTNS()
CE	Same as PAM	POF_CE_PAM()	STF_AD_FUTUR()

5.17 CEG: Credit Enhancement Guarantee

A credit-enhancement guarantee is a third-party promise to cover the losses of one or more *covered* contracts. The guarantor receives a premium at inception and, if a covered contract suffers an observed credit event, pays out the covered exposure — the sum of the covered contracts' outstanding notionals plus, for a guarantee, their accrued interest.

CEG is a composed contract: it references its covered contracts through CTST (§4) and reads their state through the child-contract observer to size the exposure at the moment of default. The payout therefore carries the guarantor's role sign and is bounded below by zero (37). CEG and the collateral variant CEC are formalized as 19.

Divergence from the reference data. Here the formalization is the arbiter. The executable engine reproduces every gated reference contract exactly, but one CEG reference case (`guarantee14`) carries a reference value that disagrees with the CEG payoff specified below — the engine computes the value this specification prescribes. We record it as a known discrepancy in the reference suite rather than a defect of the specification.

Definition 19 (CREDIT). CEG (guarantee) and CEC (collateral, §5.18) over covered contracts: a premium at purchase, and on a covered contract's observed credit event the covered exposure settles — capped at the observed collateral value for CEC.

CEG	
State Variables	
Variabl	Initialization per t_0
Md	$\mathbf{Md}_{t_0} = \begin{cases} \text{MD} & \text{if } \text{MD} \neq \emptyset \\ \max(\tau(\{e_t^k\})) & \text{else} \end{cases}$ with $\{e_t^k\} = \{e_t^{k,1}\} \cup \{e_t^{k,2}\} \cup \dots \cup \{e_t^{k,n}\}$ $\{e_t^{k,i}\} = U^{ev}(\text{CTST}_{\text{CoveredContract}}^{\text{Contract}}(i), t_0 \mid \{ \})_{n= \text{CTST}_{\text{CoveredContract}}^{\text{Contract}} }$

Nt	$\mathbf{Nt}_{t_0} = \begin{cases} 0.0 & \text{if } t_0 \geq \mathbf{Md}_{t_0} \\ \text{CECVR}(\text{CNTRL})\mathbf{NT} & \text{else if } \mathbf{NT} \neq \emptyset \\ \text{CECVR}(\text{CNTRL}) \sum_{i=1}^{ \mathbf{I} } n_i & \text{else} \end{cases}$ <p>with</p> $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{x\}) & \text{if } \text{CEGE} = \text{NO} \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{x\}) & \text{else if } \text{CEGE} = \text{NI} \\ +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{x\}) & \\ Orf(U^{ca}(I(i), \text{MOC}), t_0) & \text{else} \end{cases}$ <p>where $I = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}}$</p>		
Feac	$\mathbf{Feac}_{t_0} = \begin{cases} 0.0 & \text{if } \text{FER} = \emptyset \\ \text{FEAC} & \text{else if } \text{FEAC} \neq \emptyset \\ \mathbf{Nt}_{t_0} Y(t^-, t_0) \text{FER} & \text{else if } \text{FEB} = \text{'N'} \\ \frac{Y(t_0^{FP-}, t_0)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL}) \text{FER} & \text{else} \end{cases}$ <p>with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$</p>		
Xd	Same as OPTNS		
Xa	Same as OPTNS		
Prf	Same as PAM		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	$\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t) \mathbf{Nt} \text{FER} & \text{if } \text{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\text{CNTRL}) \text{FER} & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ <p>with $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$</p>
PRD	Same as PAM	POF_PRD_STK()	$\mathbf{Nt}' = \begin{cases} \mathbf{Nt} & \text{if } \mathbf{NT} \neq \emptyset \\ \text{CECV} \sum_{i=1}^{ \mathbf{I} } n_i & \text{else} \end{cases}$ $\mathbf{Feac}' = \mathbf{Feac}_{t_0}$ $\mathbf{Sd}' = t$ <p>with</p> $n_i = \begin{cases} U^{sv}(I(i), t, \mathbf{Nt} \mid \{\}) & \text{if } \text{CEGE} = \text{NO} \\ U^{sv}(I(i), t, \mathbf{Nt} \mid \{\}) & \text{else if } \text{CEGE} = \text{NI} \\ +U^{sv}(I(i), t, \mathbf{Ipac} \mid \{\}) & \\ Orf(U^{ca}(I(i), \text{MOC}), t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$ <p>where $I = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}}$</p>
TD	Same as PAM		

FP	$\begin{cases} \bar{t}^{FP} & \text{if } FER = \emptyset \\ \emptyset & \text{if } FER = \emptyset \\ S(s, FECL, T) & \text{else} \end{cases}$ with $\begin{cases} \emptyset & \text{if } FEANX = \emptyset \wedge FECL = \emptyset \\ PRD + FECL & \text{else if } FEANX = \emptyset \\ FEANX & \text{else} \end{cases}$ $T = \begin{cases} MD & \text{if } t^{XD} = \emptyset \\ t^{XD} & \text{else} \end{cases}$	$X_{CUR}^{CURS}(t) R(CNTRL) FER$	$\mathbf{Nt}' = \begin{cases} \mathbf{Nt} & \text{if } NT \neq \emptyset \\ \text{CECV} \sum_{i=1}^{ I } n_i & \text{else} \end{cases}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + \mathbf{Nt} Y(t^-, t) & \text{if } FER = \emptyset \\ \mathbf{Feac} & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $n_i = \begin{cases} U^{sv}(I(i), t, \mathbf{Nt} \{\}) & \text{if } CEGE = \\ U^{sv}(I(i), t, \mathbf{Nt} \{\}) & \text{else if } CEGE = \\ + U^{sv}(I(i), t, \mathbf{Ipac} \{\}) & \\ Or^f(U^{ca}(I(i), MOC), t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$ where $I = \text{CTST}_{CoveredContract}^{Contract}$
MD	$\begin{cases} t^{MD} & \\ \mathbf{Md}_{t_0} & \text{if } t^{XD} = \emptyset \\ \emptyset & \text{else} \end{cases}$	0.0	$\mathbf{Nt}' = 0.0$ $\mathbf{Sd}' = t$
XD	$\begin{cases} t^{XD} & \\ \mathbf{Xd}_{t_0} & \text{if } \mathbf{Xd}_{t_0} \neq \emptyset \\ \inf \tau(e^{CE}) & \text{else if } \tau(e^{CE}) \neq \emptyset \\ \emptyset & \text{else} \end{cases}$ with $e^{CE} = e \in O^{ev}(i, CE, t_0) \text{Prf}_{\tau(e)^+} = \text{CET} \wedge \tau(e) \leq \mathbf{Md}_{t_0}$ and $i = \text{CTST}_{CoveredContract}^{Contract}(i)$	POF_XD_OPTNS()	$\mathbf{Nt}' = \begin{cases} \mathbf{Nt} & \text{else if } NT \neq \emptyset \\ \text{CECV} \sum_{i=1}^{ I } n_i & \text{else} \end{cases}$ $\mathbf{Feac}' = \begin{cases} \mathbf{Feac} + Y(\mathbf{Sd}, t) \mathbf{Nt} FER & \text{if } FEB = \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(CNTRL) FER & \text{else} \end{cases}$ $\mathbf{Xa}' = \text{CECV} \mathbf{Nt}$ $\mathbf{Sd}' = t$ with $n_i = \begin{cases} U^{sv}(I(i), t, \mathbf{Nt} \{\}) & \text{if } CEGE = \\ U^{sv}(I(i), t, \mathbf{Nt} \{\}) & \text{else if } CEGE = \\ + U^{sv}(I(i), t, \mathbf{Ipac} \{\}) & \\ Or^f(U^{ca}(I(i), MOC), t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \bar{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \bar{t}^{FP} \mid t > t_0$ where $I = \text{CTST}_{CoveredContract}^{Contract}$
STD	Same as OPTNS	$X_{CUR}^{CURS}(t) (\mathbf{Xa} + \mathbf{Feac})$	$\mathbf{Nt}' = 0.0$ $\mathbf{Feac}' = 0.0$ $\mathbf{Sd}' = t$
CE	Same as PAM	POF_CE_PAM()	STF_AD_CEG()

5.18 CEC: Credit Enhancement Collateral

Credit-enhancement collateral is a guarantee backed by posted assets rather than a counterparty's promise: on a covered credit event the protection pays the covered exposure, but *capped* at the observed value of the collateral. The payout is thus $\min(\text{coverage} \cdot \text{exposure}, \text{collateral value})$ — it can never exceed either the covered claim or the collateral on hand.

This cap is exactly the bound proved in 37. CEC is formalized with CEG as 19.

CEC
State Variables

Variabl Initialization per t_0			
Md	$\mathbf{Md}_{t_0} = \begin{cases} \text{MD} & \text{if } \text{MD} \neq \emptyset \\ \max(\tau(\{e_t^k\})) & \text{else} \end{cases}$ with $\{e_t^k\} = \{e_t^{k,1}\} \cup \{e_t^{k,2}\} \cup \dots \cup \{e_t^{k,n}\}$ $\{e_t^{k,i}\} = U^{ev}(\text{CTST}_{\text{CoveredContract}}^{\text{Contract}}(i), t_0 \mid \{ \})n = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}} $		
Nt	$\mathbf{Nt}_{t_0} = \begin{cases} 0.0 & \text{if } t_0 \geq \mathbf{Md}_{t_0} \\ \min \left(\sum_{j=1}^{ J } v_j, \text{CECV} \sum_{i=1}^{ I } n_i \right) & \text{else} \end{cases}$ with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{if } \text{CEGE} = \text{NO} \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{else if } \text{CEGE} = \text{NI} \\ +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{ \}) \\ Orf(U^{ca}(I(i), \text{MOC}), t_0) & \text{else} \end{cases}$ $v_j = Orf(U^{ca}(J(j), \text{MOC}), t_0)$ $I = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}}$ $J = \text{CTST}_{\text{CoveringContract}}^{\text{Contract}}$		
Sd	Same as PAM		
Events			
Event	Schedule	Payoff Function	State Transition
AD	Same as PAM	POF_AD_PAM()	$\mathbf{Nt}' = \begin{cases} 0.0 & \text{if } t \geq \mathbf{Md}_{t_0} \\ \min \left(\sum_{j=1}^{ J } v_j, \text{CECV} \sum_{i=1}^{ I } n_i \right) & \text{else} \end{cases}$ $\mathbf{Sd}' = t$ with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{if } \text{CEGE} = \text{NO} \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{else if } \text{CEGE} = \text{NI} \\ +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{ \}) \\ Orf(U^{ca}(I(i), \text{MOC}), t_0) & \text{else} \end{cases}$ $v_j = Orf(U^{ca}(J(j), \text{MOC}), t_0)$ $I = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}}$ $J = \text{CTST}_{\text{CoveringContract}}^{\text{Contract}}$
XD	Same as CEG	POF_XD_OPTNS()	$\mathbf{Nt}' = \min \left(\sum_{j=1}^{ J } v_j, \text{CECV} \sum_{i=1}^{ I } n_i \right)$ $\mathbf{Sd}' = t$ with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{if } \text{CEGE} = \text{NO} \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{ \}) & \text{else if } \text{CEGE} = \text{NI} \\ +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{ \}) \\ Orf(U^{ca}(I(i), \text{MOC}), t_0) & \text{else} \end{cases}$ $v_j = Orf(U^{ca}(J(j), \text{MOC}), t_0)$ $I = \text{CTST}_{\text{CoveredContract}}^{\text{Contract}}$ $J = \text{CTST}_{\text{CoveringContract}}^{\text{Contract}}$
STD	Same as CEG	$X_{\text{CUR}}^{\text{CURS}}(t)\mathbf{Nt}$	$\mathbf{Nt}' = 0.0$ $\mathbf{Sd}' = t$
MD	Same as CEG	0.0	$\mathbf{Nt}' = 0.0$ $\mathbf{Sd}' = t$

5.19 Schedule generation and execution

Definition 20 (Event schedule generation). `genSchedule` builds the full event `Schedule` (IED, IP/IPCI, PR, RR/RRF, FP, SC, MD, ...) for a stateful contract from its dictionary terms, following the Contract-Schedule tables above; `runSchedule` then folds the contract's `stf/pof` over it. This is the type-agnostic core shared by every contract family.

6 Metatheorems

The formalization adds guarantees the original specification never states: the relational and functional models agree, status dates advance monotonically, maturity zeroes the notional, and — over \mathbb{R} — a battery of quantitative bounds and conservation laws hold of every execution.

Theorem 21 (Relational/functional agreement). *For each of the eight stateful contracts (PAM, LAM, NAM, ANN, CLM and LAX, SWPPV, UMP) every relational step lands on the functional next-state for some event and time; conversely every admissible event/time yields a step. Each relation is exactly the graph of its `stf`.*

Theorem 22 (Status-date monotonicity). *Along any execution trace the status date is non-decreasing: each step requires $Sd \leq t$ and sets $Sd := t$.*

Theorem 23 (Maturity). *The maturity event zeroes the notional principal (and accrued interest and fees); termination additionally zeroes the interest rate.*

6.1 Quantitative bounds and conservation (over \mathbb{R})

Theorem 24 (Rate cap/floor bound). *After a rate reset the nominal rate lies within the life floor/cap window $[RRLF, RRLC]$ (whenever both are present and well-ordered). Holds for PAM, for ANN (by reuse of PAM's rate logic), and for SWPPV's floating-rate reset. With every cap/floor absent the reset instead returns the raw repricing target $O^{rf}(RRMO, t) \cdot RRMLT + RRSP$ (`pam_rr_uncapped`).*

Theorem 25 (Redemption non-overshoot). *A principal redemption never overshoots: with a sign-normalised nonnegative state the post-redemption notional stays in $[0, Nt]$, and the redeemed amount never exceeds the outstanding notional in magnitude ($|\text{redeemed}| \leq |Nt|$).*

Theorem 26 (Strict amortization and full repayment). *With a strictly positive notional and instalment a redemption strictly decreases the notional (guaranteed progress); once the instalment reaches the outstanding notional the step drives it to exactly 0 (clean repayment).*

Theorem 27 (Interest capitalization conserves value). *The IPCI event moves accrued interest into the notional and resets accrued interest to 0: value is relocated, not created. (This one holds definitionally at any amount type.)*

Definition 28 (Principal-redemption trace). `PRTrace` is the sub-relation of the LAM execution trace consisting of zero or more admissible principal-redemption (PR) steps — the regime in which the notional should be conserved (events like IED/IPCI reset or grow it).

Theorem 29 (Trace-level conservation of the notional). *Along an entire principal-redemption trace the notional stays in $[0, Nt_0]$ and never increases in magnitude — the single-step non-overshoot lifted to whole executions by induction over the trace. Every `PRTrace` is moreover a genuine LAM execution trace, so this is a statement about real runs.*

Definition 30 (Capitalization trace). `IPCITrace` is the chain of zero or more admissible interest capitalizations (IPCI) of a UMP deposit; `ipciFactor` accumulates the multiplicative growth factor $\prod_i (1 + \text{rate} \cdot Y_i)$ along it.

Theorem 31 (UMP compound-growth closed form). *Along a capitalization trace the notional grows multiplicatively to $Nt' = Nt_0 \cdot \prod_i (1 + \text{rate} \cdot Y_i)$; the nominal rate is invariant under IPCI (justifying the single rate), and with a non-negative rate and year fraction each capitalization never shrinks the deposit.*

Definition 32 (LAX redemption trace). `LAXPRTrace` is the chain of zero or more admissible LAX principal redemptions (PR) with non-negative instalments.

Theorem 33 (LAX monotone amortization). *On the long side (sign ≥ 0) a LAX redemption with a non-negative instalment never increases the notional, so along an entire redemption trace $Nt' \leq Nt_0$ (monotone repayment). Unlike LAM's capped `redeemed` this is a one-sided bound, since `STF_PR_LAX` subtracts the raw instalment. Every such trace is a genuine LAX execution.*

Theorem 34 (Swap structural invariants). *A swap exchanges interest, not principal: no `SWPPV` event moves the notional, and only a rate reset (`RR`) changes the floating rate. Status dates are non-decreasing along any execution (even though the fixed leg and reset leave `Sd` untouched and only the floating leg advances it).*

6.2 Payoff bounds for the derivative and credit families (over \mathbb{R})

Theorem 35 (Cap/floor and option intrinsics are non-negative). *The cap/floor rate-excess kernel $\max(r - \text{cap}, 0) + \max(\text{floor} - r, 0)$ and the European call/put intrinsics $\max(S - K, 0)$, $\max(K - S, 0)$ are all ≥ 0 , so a long cap, floor, or option never settles negative; a call is in the money exactly when the underlying exceeds the strike.*

Theorem 36 (Settlement conservation for FX and swaps). *An FX outright's two delivery legs net to the par difference $R \cdot (Nt_1 - Nt_2)$ at unit FX; a swap's per-period fixed and floating legs net to $R \cdot N \cdot (\text{fix} - \text{flt}) \cdot Y$ — exactly what the net-settlement fold computes from the two gross legs. For `SWPPV` this is now proven on the actual `stf/pof` state machine (`swppv_pof_net`), and at par (`fix = flt`) the netted period flow is zero.*

Theorem 37 (Credit-enhancement payout bounds). *A collateral payout $\min(\text{coverage-exposure}, \text{value})$ never exceeds either the posted collateral value or the covered claim; the covered exposure, a sum of per-leg magnitudes, is non-negative, so the payout carries the contract-role sign.*

7 Data Dictionary

The tables below reproduce the *ACTUS Data Dictionary* (ACTUS Financial Research Foundation, v1.4, CC-BY-SA-4.0), the normative glossary of the standard. They explain every contract-term, state-variable and contract-type acronym used throughout this document; the lifecycle event types (IED, IP, PR, ...) are not part of the dictionary and so are not listed here. Acronyms in `typewriter` font elsewhere in the blueprint link to their entry below.

7.1 Contract Terms

Acronym	Name	Description
AMD	Amortization Date	This Date is used to calculate the annuity amounts for ANN and ANX NGX CT's. Needs only to be set in case where the contract balloon at MD and MD is less than AD.
ARFIXVAR	Array Fixed Variable	For array-type rate reset schedules, this attributes defines the meaning of ARRATE.
ARINCDEC	Array Increase Decrease	Indicates whether a certain PRNXT element in ARPRNX increases the principal (NT) or decreases it. Applies only for ANX, NAX, LAX Maturity CTs. For all other Maturity CTs the first principal payment is always in the opposite direction of all other (following) payments.
ARIPANXi	Array Cycle Anchor Date Of Interest Payment	Same like IPANX but as array
ARIPCLi	Array Cycle Of Interest Payment	Same like IPCL but as array
ARPRANXj	Array Cycle Anchor Date Of Principal Redemption	Same like PRANX but as array
ARPRCLj	Array Cycle Of Principal Redemption	Same like PRCL but as array
ARPRNXTj	Array Next Principal Redemption Payment	Same like PRNXT but as array
ARRATE	Array Rate	For array-type rate reset schedules, this attribute represents either an interest rate (corresponding to IPNR) or a spread (corresponding to RRSP). Which case applies depends on the attribute ARFIXVAR: if ARFIXVAR=FIX then it represents the new IPNR and if ARFIXVAR=VAR then the applicable RRSP.
ARRRANX	Array Cycle Anchor Date Of Rate Reset	Same like RRANX but as array
ARRRCL	Array Cycle Of Rate Reset	Same like RRCL but as array
BCF	Boundary Crossed Flag	Initializes the value of Boundary Crossed Flag state variable at statusDate

Acronym	Name	Description
BDC	Business Day Convention	BDC's are linked to a calendar. Calendars have working and non-working days. A BDC value other than N means that cash flows cannot fall on non-working days, they must be shifted to the next business day (following) or the previous one (preceding). These two simple rules get refined twofold: - Following modified (preceding): Same like following (preceding), however if a cash flow gets shifted into a new month, then it is shifted to preceding (following) business day. - Shift/calculate (SC) and calculate/shift (CS). Accrual, principal, and possibly other calculations are affected by this choice. In the case of SC first the dates are shifted and after the shift cash flows are calculated. In the case of CS it is the other way round. Attention: Does not affect non-cyclical dates such as PRD, MD, TD, IPCED since they can be set to the correct date directly.
BDR	Boundary Direction	Boundary direction specifies the direction of motion in the underlying asset's price which will be considered a valid crossing of the boundary and trigger the boundary effect changing which, if any, of the boundary legs is active.
BEF	Boundary Effect	This term specifies which leg - if any- becomes the active subcontract when the underlying asset's price crosses the specified boundary value in the specified direction triggering a boundary crossing event.
BLIA	Boundary Leg Initially Active	Specifies which leg - if any - is the active contract in effect when the boundary controlled switch contract starts.
BMANX	Boundary Monitoring Anchor Date	The first Boundary monitoring event occurs on this date
BMCL	Boundary Monitoring Cycle	The frequency with which boundary monitoring events occur. It defines how often the system checks to test whether the market value of the underlying asset has crossed the boundary in the specified direction triggering a boundary crossing event.
BMED	Boundary Monitoring End Date	Boundary monitoring ends on this date
BV	Boundary Value	Boundary value in a barrier options contract, when reached, triggers the boundary effect specified e.g. Knock-In or Knock-out
CDD	Contract Deal Date	This date signifies the origination of the contract where an agreement between the customer and the bank has been settled. From this date on, the institution will have a (market) risk position for financial contracts. This is even the case when IED is in future.
CECV	Coverage Of Credit Enhancement	Defines which percentage of the exposure is covered
CEGE	Guaranteed Exposure	Defines which value of the exposure is covered: - NO: Nominal Value - NI: Nominal plus Interest - MV: Market Value

Acronym	Name	Description
CETC	Credit Event Type Covered	The type of credit events covered e.g. in credit enhancement or credit default swap contracts. Only the defined credit event types may trigger the protection.
CID	Contract Identifier	Unique identifier of a contract. If the system is used on a single firm level, an internal unique ID can be generated. If used on a national or globally level, a globally unique ID is required.
CLA	Credit Line Amount	If defined, gives the total amount that can be drawn from a credit line. The remaining amount that can still be drawn is given by CLA-NT. For ANN, NAM, the credit line can only be drawn prior to PRANX-1PRCL. For CRL, the remaining amount that can still be drawn is given by CLA-Sum(NT of attached contracts).
CLDR	Calendar	Calendar defines the non-working days which affect the dates of contract events (CDE's) in combination with EOMC and BDC. Custom calendars can be added as additional enum options.
CNTRL	Contract Role	CNTRL defines which position the CRID (the creator of the contract record) takes in a contract. For example, whether the contract is an asset or liability, a long or short position for the CRID. Most contracts are simple on or off balance sheet positions which are assets, liabilities. Such contracts can also play a secondary role as a collateral. The attribute is highly significant since it determines the direction of all cash flows. The exact meaning is given with each CT in the ACTUS High Level Specification document.
CPID	Counterparty Identifier	CPID identifies the counterparty to the CRID in this contract. CPID is ideally the official LEI which can be a firm, a government body, even a single person etc. However, this can also refer to a anonymous group in which case this information is not to be disclosed. CPID may also refer to a group taking a joint risk or more generally, CPID is the main counterparty, against which the contract has been settled.
CRID	Creator Identifier	This identifies the legal entity creating the contract record. The counterparty of the contract is tracked in CPID. CRID is ideally the official LEI which can be a firm, a government body, even a single person etc. However, this can also refer to a anonymous group in which case this information is not to be disclosed. CRID may also refer to a group taking a joint risk.
CT	Contract Type	The ContractType is the most important information. It defines the cash flow generating pattern of a contract. The ContractType information in combination with a given state of the risk factors will produce a deterministic sequence of cash flows which are the basis of any financial analysis.

Acronym	Name	Description
CTS	Contract Structure	A structure identifying individual or sets of underlying contracts. E.g. for FUTUR, this structure identifies the single underlying contract, for SWAPS, the FirstLeg and Second-Leg are identified, or for CEG, CEC the structure identifies Covered and Covering contracts.
CUR	Currency	The currency of the cash flows.
CUR2	Currency 2	The currency of the cash flows of the second leg (if not defined, main currency applies)
CURS	Settlement Currency	The currency in which cash flows are settled. This currency can be different from the currency (CUR) in which cash flows or the contract, respectively, is denominated in which case the respective FX-rate applies at settlement time. If no settlement currency is defined the cash flows are settled in the currency in which they are denominated.
DQP	Delinquency Period	If real payment happens after scheduled payment date plus DLP, then the counterparty is in technical default. This means that the creditor legally has the right to declare default of the debtor.
DQR	Delinquency Rate	Rate at which Delinquency Payments accrue on NT (in addition to the interest rate) during the DelinquencyPeriod
DS	Delivery Settlement	Indicates whether the contract is settled in cash or physical delivery. In case of physical delivery, the underlying contract and associated (future) cash flows are effectively exchanged. In case of cash settlement, the current market value of the underlying contract determines the cash flow exchanged.
DVANX	Cycle Anchor Date Of Dividend	Date from which the dividend payment date schedule is calculated according to the cycle length. The first dividend payment event takes place on this anchor.
DVCL	Cycle Of Dividend	Defines in combination with DVANX the payment points of dividends. The dividend payment schedule will start at DVANX and end at MaximumProjectionPeriod (cf. sheet Modeling Parameters).
DVEX	Ex Dividend Date	In case contract is traded between DVEX and next DV payment date (i.e. $PRD > DVEX$ & $PRD < \text{next DV payment date}$), then the old holder of the contract (previous to the trade) receives the next DV payment. In other words, the next DV payment is cancelled for the new (after the trade) holder of the contract.
DVNP	Next Dividend Payment Amount	Defines the next dividend payment (amount) whereas the date of dividend payment is defined through the DVANX/DVCL pair. If DVCL is defined, then this amount will be used as dividend payment for each future dividend payment date.

Acronym	Name	Description
EOMC	End Of Month Convention	When computing schedules a special problem arises if an anchor date is at the end of a month and a cycle of monthly or quarterly is applied (yearly in the case of leap years only). How do we have to interpret an anchor date April 30 plus 1M cycles? In case where EOM is selected, it will jump to the 31st of May, then June 30, July 31 and so on. If SM is selected, it will jump to the 30st always with of course an exception in February. This logic applies for all months having 30 or less days and an anchor date at the last day. Month with 31 days will at any rate jump to the last of the month if anchor date is on the last day.
FEAC	Fee Accrued	Accrued fees as per SD
FEANX	Cycle Anchor Date Of Fee	Date from which the fee payment date schedule is calculated according to the cycle length. The first fee payment event takes place on this anchor.
FEB	Fee Basis	Basis, on which Fee is calculated. For FEB='A', FER is interpreted as an absolute amount to be paid at every FP event and for FEB='N', FER represents a rate at which FP amounts accrue on the basis of the contract's NT.
FECL	Cycle Of Fee	Defines in combination with FEANX the payment points of fees
FER	Fee Rate	Rate of Fee which is a percentage of the underlying or FER is an absolute amount. For all contracts where FEB does not apply (cf. business rules), FER is interpreted as an absolute amount.
GRP	Grace Period	If real payment happens after scheduled payment date plus GRP, then the payment is in delay.
IED	Initial Exchange Date	Date of the initial cash flow for Maturity and Non-Maturity CT's. It also coincides with the beginning of interest accrual calculation.
IPAC	Accrued Interest	Accrued interest as per SD. In case of NULL, this value will be recalculated using IPANX, IPCL and IPNR information. Can be used to represent irregular next IP payments.
IPANX	Cycle Anchor Date Of Interest Payment	Date from which the interest payment date schedule is calculated according to the cycle length. The first interest payment event takes place on this anchor.
IPCB	Interest Calculation Base	This is important for amortizing instruments. The basis of interest calculation is normally the notional outstanding amount as per SD. This is considered the fair basis and in many countries the only legal basis. If NULL or NTSD is selected, this is the case. Alternative bases (normally in order to favor the lending institution) are found. In the extreme case the original balance (PCDD=NT+PDCDD) never gets adjusted. In this case PCDD must be chosen. An intermediate case exist wherre balances do get adjusted, however with lags. In this case NTL mut be selected and anchor dates and cycles must be set.

Acronym	Name	Description
IPCBA	Interest Calculation Base Amount	This is the amount used for the calculation of interest. Calculation base per SD.
IPCBANX	Cycle Anchor Date Of Interest Calculation Base	Date from which the interest calculation base date schedule is calculated according to the cycle length. The first interest calculation base event takes place on this anchor.
IPCACL	Cycle Of Interest Calculation Base	Concerning the format see PRCL. Defines the subsequent adjustment points to NT of the interest payment calculation base.
IPCED	Capitalization End Date	If IPCED is set, then interest is not paid or received but added to the balance (NT) until IPCED. If IPCED does not coincide with an IP cycle, one additional interest payment gets calculated at IPCED and capitalized. Thereafter normal interest payments occur.
IPCL	Cycle Of Interest Payment	Cycle according to which the interest payment date schedule will be calculated. In case IPCL is not set, then there will only be an interest payment event at MD (and possibly at IPANX if set). The interval will be adjusted yet by EOMC and BDC.
IPDC	Day Count Convention	Method defining how days are counted between two dates. This finally defines the year fraction in accrual calculations.
IPNR	Nominal Interest Rate	The nominal interest rate which will be used to calculate accruals and the next interest payment at the next IP date. NT multiplied with IPNR is the base for the interest payment calculation. The relevant time period is a function of IPDC. If the contract is variable (RRANX set) this field is periodically updated per SD. In the case of plan vanilla interest rate swaps (IRSPV) this defines the rate of fixed leg.
IPNR2	Nominal Interest Rate 2	The nominal interest rate which will be used to calculate accruals and the next interest payment at the next IP date on the second leg (the one not mentioned in CNTRL) of a plain vanilla swap. The relevant time period is a function of IPDC. It is periodically updated per SD.
IPPNT	Cycle Point Of Interest Payment	Usually, interest is paid at the end of each IPCL which corresponds to a IPPNT value of E which is also the default. If interest payment occurs at the beginning of the cycle, the value is B.
MD	Maturity Date	Marks the contractual end of the lifecycle of a CT. Generally, date of the last cash flows. This includes normally a principal and an interest payment. Some Maturity CTs as perpetuals (PBN) do not have such a date. For variable amortizing contracts of the ANN CT, this date might be less than the scheduled end of the contract (which is deduced from the periodic payment amount PRNXT). In this case it balloons.
MOC	Market Object Code	Is pointing to the market value at SD (MarketObject). Unique codes for market objects must be used.

Acronym	Name	Description
MPFD	Maximum Penalty Free Disbursement	Defines the notional amount which can be withdrawn before XDN without penalty
MRANX	Cycle Anchor Date Of Margining	Date from which the margin call date schedule is calculated according to the cycle length. The first margin call event takes place on this anchor.
MRCL	Cycle Of Margining	Defines together with MRANX the points where margins can be called.
MRCLH	Clearing House	Indicates wheter CRID takes a clearing house function or not. In other word, whether CRID receive margins (MRIM, MRVM).
MRIM	Initial Margin	Margin to cover losses which may be incurred as a result of market fluctuations. Upon contract closing or maturity, the MRIM is reimbursed.
MRMML	Maintenance Margin Lower Bound	Defines the lower bound of the Maintenance Margin. If MRVM falls below MRMML, then capital must be added to reach the original MRIM.
MRMMU	Maintenance Margin Upper Bound	Defines the upper bound of the Maintenance Margin. If MRVM falls above MRMMU, then capital is refunded to reach the original MRIM.
MRVM	Variation Margin	MRVM reflects the accrued but not yet paid margin as per SD. Open traded positions are revalued by the exchange at the end of every trading day using mark-to-market valuation. Often clearing members do not credit or debit their clients daily with MRVM, but rather use a Maintenance Margin. If the balance falls outside MRMML (and MRMMU), then capital must be added (is refunded) to reach the original margin amount MRIM. We can also say that MVO+MRVM is equal to the reference value as per last margin update.
MVO	Market Value Observed	Value as observed in the market at SD per unit. In case of fixed income instruments it is a fraction.
NPD	Non Performing Date	The date of the (uncovered) payment event responsible for the current value of the Contract Performance attribute.
NT	Notional Principal	Current nominal value of the contract. For debt instrument this is the current remaining notional outstanding. NT is generally the basis on which interest payments are calculated. If IPCBS is set, IPCBS may introduce a different basis for interest payment calculation.
NT2	Notional Principal 2	Notional amount of the second currency to be exchanged in an FXOUT CT.
OPANX	Cycle Anchor Date Of Optionality	Used for Basic Maturities (such as PAM, RGM, ANN, NGM and their Step-up versions) and American and Bermudan style options. - Basic Maturities: Within the group of these Maturities, it indicates the possibility of prepayments. Prepayment features are controlled by Behavior. - American and Bermudan style Options: Begin of exercise period.

Acronym	Name	Description
OPCL	Cycle Of Optionality	Cycle according to which the option exercise date schedule will be calculated. OPCL can be NULL for American Options or Prepayment Optionality in which case the optionality period starts at OPANX and ends at OPXED (for american options) or MD (in case of prepayment optionality). The interval will be adjusted yet by EOMC and BDC.
OPS1	Option Strike 1	Strike price of the option. Whether it is a call/put is determined by the attribute OPTP, i.e a call or a put (or a combination of call/put). This attribute is used for price related options such as options on bonds, stocks or FX. Interest rate related options (caps/floors) are handled within the RatReset group.
OPTP	Option Type	Defines whether the option is a call or put or a combination of it. This field has to be seen in combination with CNTRL where it is defined whether CRID is the buyer or the seller.
OPXED	Option Exercise End Date	Final exercise date for American and Bermudan options, expiry date for European options.
OPXT	Option Exercise Type	Defines whether the option is European (exercised at a specific date), American (exercised during a span of time) or Bermudan (exercised at certain points during a span of time).
PDIED	Premium Discount At IED	Total original premium or discount that has been set at CDD and will be added to the (notional) cash flow at IED (cash flow at IED = NT+PDIED, w.r.t. an RPA CT). Negative value for discount and positive for premium. Note, similar to interest the PDIED portion is part of P&L.
PFUT	Futures Price	The price the counterparties agreed upon at which the underlying contract (of a FUTUR) is exchanged/settled at STD. Quoting is different for different types of underlyings: Fixed Income = in percentage, all others in nominal terms.
PPEF	Prepayment Effect	This attribute defines whether or not the right of prepayment exists and if yes, how prepayment affects the remaining principal redemption schedule of the contract
PPP	Prepayment Period	If real payment happens before scheduled payment date minus PPP, then it is considered a prepayment. Effect of prepayments are further described in PPEF and related fields.
PPRD	Price At Purchase Date	Purchase price exchanged at PRD. PPRD represents a clean price (includes premium/discount but not IPAC).
PRANX	Cycle Anchor Date Of Principal Redemption	Date from which the principal payment date schedule is calculated according to the cycle length. The first principal payment event takes place on this anchor.
PRCL	Cycle Of Principal Redemption	Cycle according to which the interest payment date schedule will be calculated. In case PRCL is not set, then there will only be one principal payment event at MD (and possibly at PRANX if set). The interval will be adjusted yet by EOMC and BDC.

Acronym	Name	Description
PRD	Purchase Date	If a contract is bought after initiation (for example a bond on the secondary market) this date has to be set. It refers to the date at which the payment (of PPRD) and transfer of the security happens. In other words, PRD - if set - takes the role otherwise IED has from a cash flow perspective. Note, CPID of the CT is not the counterparty of the transaction!
PRF	Contract Performance	Indicates the current contract performance status. Different states of the contract range from performing to default.
PRNXT	Next Principal Redemption Payment	Amount of principal that will be paid during the redemption cycle at the next payment date. For amortizing contracts like ANN, NAM, ANX, and NAX this is the total periodic payment amount (sum of interest and principal).
PTD	Price At Termination Date	Selling price exchanged at PTD PTD represents a clean price (includes premium/discount but not IPAC)
PYRT	Penalty Rate	Either the rate or the absolute amount of the prepayment.
PYTP	Penalty Type	Defines whether prepayment is linked to a penalty and of which kind.
QT	Quantity	This attribute relates either to physical contracts (COM) or underlyings of traded contracts. In case of physical contracts it holds the number of underlying units of the specific good (e.g. number of barrels of oil). In case of well defined traded contracts it holds the number of defined underlying instruments. Example: QT of STK CTs underlying a FUTUR indicates the number of those specific STK CTs which underlie the FUTUR.
RRANX	Cycle Anchor Date Of Rate Reset	Date from which the rate reset date schedule is calculated according to the cycle length. The first rate reset event takes place on this anchor.
RRCL	Cycle Of Rate Reset	Cycle according to which the rate reset date schedule will be calculated. In case RRCL is not set, then there will only be one rate reset event at RRANX given RRANX if set. The interval will be adjusted yet by EOMC and BDC.
RRFIX	Fixing Period	Interest rate resets (adjustments) are usually fixed one or two days (usually Business Days) before the new rate applies (defined by the rate reset schedule). This field holds the period between fixing and application of a rate.
RRLC	Life Cap	For variable rate basic CTs this represents a cap on the interest rate that applies during the entire lifetime of the contract. For CAPFL CTs this represents the cap strike rate.
RRLF	Life Floor	For variable rate basic CTs this represents a floor on the interest rate that applies during the entire lifetime of the contract. For CAPFL CTs this represents the floor strike rate.

Acronym	Name	Description
RRMLT	Rate Multiplier	Interest rate multiplier. A typical rate resetting rule is LIBOR plus x basis point where x represents the interest rate spread. However, in some cases like reverse or super floater contracts an additional rate multiplier applies. In this case, the new rate is determined as: IPNR after rate reset = Rate selected from the market object * RRMLT + RRSP.
RRMO	Market Object Code Of Rate Reset	Is pointing to the interest rate driver (MarketObject) used for rate reset uniquely. Unique codes for market objects must be used.
RRNXT	Next Reset Rate	Holds the new rate that has been fixed already (cf. attribute FixingDays) but not applied. This new rate will be applied at the next rate reset event (after SD and according to the rate reset schedule). Attention, RRNXT must be set to NULL after it is applied!
RRPC	Period Cap	For variable rate basic CTs this represents the maximum positive rate change per rate reset cycle.
RRPF	Period Floor	For variable rate basic CTs this represents the maximum negative rate change per rate reset cycle.
RRPNT	Cycle Point Of Rate Reset	Normally rates get reset at the beginning of any resetting cycles. There are contracts where the rate is not set at the beginning but at the end of the cycle and then applied to the previous cycle (post-fixing); in other words the rate applies before it is fixed. Hence, the new rate is not known during the entire cycle where it applies. Therefore, the rate will be applied backwards at the end of the cycle. This happens through a correction of interest accrued.
RRSP	Rate Spread	Interest rate spread. A typical rate resetting rule is LIBOR plus x basis point where x represents the interest rate spread. The following equation can be taken if RRMLT is not set: IPNR after rate reset = Rate selected from the market object + RRSP.
SCANX	Cycle Anchor Date Of Scaling Index	Date from which the scaling date schedule is calculated according to the cycle length. The first scaling event takes place on this anchor.
SCDD	Scaling Index At Contract Deal Date	The value of the Scaling Index as per Contract Deal Date.
SCCL	Cycle Of Scaling Index	Cycle according to which the scaling date schedule will be calculated. In case SCCL is not set, then there will only be one scaling event at SCANX given SCANX is set. The interval will be adjusted yet by EOMC and BDC.
SCEF	Scaling Effect	Indicates which payments are scaled. I = Interest payments, N = Nominal payments and M = Maximum deferred interest amount. They can be scaled in any combination.
SCIP	Interest Scaling Multiplier	The multiplier being applied to interest cash flows

Acronym	Name	Description
SCMO	Market Object Code Of Scaling Index	
SCNT	Notional Scaling Multiplier	The multiplier being applied to principal cash flows
SD	Status Date	SD holds the date per which all attributes of the record were updated. This is especially important for the highly dynamic attributes like Accruals, Notional, interest rates in variable instruments etc.
SEN	Seniority	Refers to the order of repayment in the event of a sale or default of the issuer.
STP	Settlement Period	Defines the period from fixing of a contingent event/obligation (Exercise Date) to settlement of the obligation.
TD	Termination Date	If a contract is sold before MD (for example a bond on the secondary market) this date has to be set. It refers to the date at which the payment (of PTD) and transfer of the security happens. In other words, TD - if set - takes the role otherwise MD has from a cash flow perspective. Note, CPID of the CT is not the counterparty of the transaction!
UT	Unit	The physical unit of the contract. Example: Barrels for an Oil COM CT.
XA	Exercise Amount	The amount fixed at Exercise Date for a contingent event/obligation such as a forward condition, optionality etc. The Exercise Amount is fixed at Exercise Date but not settled yet.
XD	Exercise Date	Date of exercising a contingent event/obligation such as a forward condition, optionality etc. The Exercise date marks the observed timestamp of fixing the contingent event and respective payment obligation not necessarily the timestamp of settling the obligation.
XDN	X Day Notice	Used as rolling attribute with the callable CT's UMP and CLM uniquely. CLM's and UMP's will not be settled (MD not set) until the client uses his option to call the contract X_Day_Notice after Current Date. As long as MD or TD is not set, the client postpones his right to call to the future. The cycle is normally defined in number of business days.
SSS	Option Strike 2	Put price in case of call/put.

7.2 State Variables

Acronym	Name	Description
Bcf	Boundary Crossed Flag	TRUE value indicates that a boundary crossing event has already occurred; boundary monitoring is no longer in effect.
Blaf1	Boundary Leg1 Active Flag	TRUE value indicates that boundaryLeg1 is currently active and generating cashflow events.

Acronym	Name	Description
Blaf2	Boundary Leg2 Active Flag	TRUE value indicates that boundaryLeg2 is currently active and generating cashflow events.
Feac	Fee Accrued	The current value of accrued fees
Icba	Interest Calculation Base Amount	The basis at which interest is being accrued. Potentially different from NVL.
Ipac	Accrued Interest	The current value of accrued interest
Ipac2	Accrued Interest 2	The current value of accrued interest of the second leg
Ipnr	Nominal Interest Rate	The applicable nominal rate
Ipnr2	Nominal Interest Rate 2	The applicable nominal rate
Md	Maturity Date	The timestamp as per which the contract matures according to the initial terms or as per unscheduled events
Npd	Non Performing Date	The date of the (uncovered) payment event responsible for the current value of the Contract Performance state variable.
Nt	Notional Principal	The outstanding nominal value
Nt2	Notional Principal 2	The outstanding nominal value of the second leg
Prf	Contract Performance	Indicates the current Contract Performance status. Different states of the contract range from performing to default.
Prnxt	Next Principal Redemption Payment	The value at which principal is being repaid. This may be including or excluding of interest depending on the Contract Type
Scip	Interest Scaling Multiplier	The multiplier being applied to interest cash flows
Scnt	Notional Scaling Multiplier	The multiplier being applied to principal cash flows
Sd	Status Date	The timestamp as per which the state is captured at any point in time
Td	Termination Date	The timestamp of unscheduled termination of a contract
Xa	Exercise Amount	The amount fixed for a the contingent event/obligation exercised at Exercise Date
Xd	Exercise Date	The timestamp at which a contingent event/obligation is exercised

7.3 Contract Types

Acronym	Name	Description
ANN	Annuity	Principal payment fully at IED and interest plus principal repaid periodically in constant amounts till MD. If variable rate, total amount for interest and principal is recalculated to be fully matured at MD.

Acronym	Name	Description
ANX	Exotic Annuity	Exotic version of ANN However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.
BCS	Boundary Controlled Switch	A boundary controlled switch is a derivative contract with subcontract legs which can be activated (knocked in) or extinguished (knocked out) when the underlying asset price reaches a specified value. The underlying asset may be a stock, index, or exchange-traded fund. Boundary controlled switch contracts with a single boundary are currently defined.
BNDCP	Convertible Note	Bonds with a call or put option. If option is exercised, underlying bond ceases to exist.
BNDWR	Warrant	Bonds with a warrant. If option is exercised, underlying bond continues to exist.
CAPFL	Cap Floors	Interest rate option expressed in a maximum or minimum interest rate.
CDSWP	Credit Default Swap	A payment is triggered if one or more of the underlying counterparties defaults.
CEC	Collateral	Collateral creates a relationship between a collateral obligee and a debtor, covering the exposure from the debtor with the collateral.
CEG	Guarantee	Guarantee creates a relationship between a guarantor, an obligee and a debtor, moving the exposure from the debtor to the guarantor.
CLM	Call Money	Loans that are rolled over as long as they are not called. Once called it has to be paid back after the stipulated notice period.
CLNTE	Credit Linked Note	A credit linked note is a security with an embedded CDSWP
COM	Commodity	This is not a financial contract in its proper sense. However it tracks movements of commodities such as oil, gas or even houses. Such commodities can serve as underlyings of commodity futures, guarantees or simply asset positions.
CSH	Cash	Cash or cash equivalent position
EXOTi	EXOTi	As of current, most of the exotic options which were popular before 2008 are out of favor and factually irrelevant. Which of the exotic options will be implemented will depend on the real relevance in the future.
FUTUR	Future	Keeps track of value changes for any basic CT as underlying (PAM, ANN etc. but also FXOUT, STK, COM). Handles margining calls.
FXOUT	Foreign Exchange Outright	Two parties agree to exchange two fixed cash flows in different currencies at a certain point in time in future.

Acronym	Name	Description
LAM	Linear Amortizer	Principal payment fully at IED. Principal repaid periodically in constant amounts till MD. Interest gets reduced accordingly. If variable rate, only interest payment is recalculated. Fixed and variable rates.
LAX	Exotic Linear Amortizer	Exotic version of LAM. However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.
MAR	Margining	A Margining contract traces the value changes and the different margin categories like initial and variation margin.
NAM	Negative Amortizer	Similar as ANN. However when resetting rate, total amount (interest plus principal) stay constant. MD shifts. Only variable rates.
NAX	Exotic Negative Amortizer	Exotic version of NAM However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.
OPTNS	Option	Calculates straight option pay-off for any basic CT as underlying (PAM, ANN etc.) but also SWAPS, FXOUT, STK and COM. Single, periodic and continuous strike is supported.
PAM	Principal at Maturity	Principal payment fully at Initial Exchange Date (IED) and repaid at Maturity Date (MD). Fixed and variable rates.
PBN	Perpetual Bonds	Bonds without any maturity date. Interest is paid into eternity if is not terminated.
REP	Repurchase Agreement	A Repo contract controls and manages the sale and repurchase of assets on the books.
SCRCR	Securitization Credit Risk	Securitization contracts where contracts are ranked according to credit default. The lower ranked tranches are hit by the first defaults. Only when the lowest tranches are wiped out, the next higher tranche is hit.
SCMR	Securitization Market Risk	Securitization contracts where all underlying contracts are treated equal. The buyer of a tranche gets a part of the cash-flows.
STK	Stock	Any instrument which is bought at a certain amount (market price normally) and then follows an index.
SWAPS	Swap	Exchange of two basic CT's (PAM, ANN etc.). Normally one is fixed, the other variable. However all variants possible including different currencies for cross currency swaps, basic swaps or even different principal exchange programs.
SWPPV	Plain Vanilla Swap	Plain vanilla swaps where the underlyings are always two identical PAM's however with one leg fixed and the other variable.

Acronym	Name	Description
TRSWP	Total Return Swap	A total return swap is a swap agreement in which one party makes payments based on a set rate, either fixed or variable, while the other party makes payments based on the return of an underlying asset, which includes both the income it generates and any capital gains.
UMP	Undefined Maturity Profile	Principal paid in and out at any point in time without pre-fixed schedule. Interest calculated on outstanding and capitalized periodically. Needs link to a behavioral function describing expected flows.